

SMDS-based Rigid Body Localization

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Abstract—We consider a novel rigid body localization (RBL) method, based only on a set of measurements of the distances, as well as the angles between sensors of the vehicle to the anchor landmark points. A key point of the proposed method is to use a variation of the super multidimensional scaling (SMDS) algorithm, where only a minor part of the complex edge kernel is used, based on the available information, which in the case of RBL is anchor-to-anchor and target-to-target information. Simulation results illustrate the good performance of the proposed technique in terms of mean square error (MSE) of the estimates, compared also to the corresponding Cramér-Rao Lower Bound (CRLB).

Index Terms—Rigid Body Localization, Multidimensional Scaling, Cramér-Rao Lower Bound, Heterogenous Information.

I. INTRODUCTION

Wireless localization [1] can be seen as one of the main applications in beyond fifth-generation (B5G) and sixth-generation (6G) systems, in light of the goals defined in IMT-2030 [2] demonstrating how users can be localized using only a radio signal, which in fact can be a communication signal used for joint communication and sensing (JCAS) applications. While there are many types of information that can be extracted from radio signals for the purpose of localization, including finger-prints [3], received signal strength indicator (RSSI) [4], angle of arrival (AoA) [5], or delay-based estimates of radio range [6], with the large demand of high accuracy localization techniques, one has to consider the combination of different types of information to improve the performance. To that extend, [7], [8] proposed the SMDS algorithm as an extension of the conventional multidimensional scaling (MDS) method [9], [10], which localizes independent target nodes by a combination of distance and angle measurements.

A consequence of this development and the large amount of new application is an increasing interest in the rigid body localization (RBL) problem [11]–[14], whose objective is to determine not only the location of targets, but also their shape and orientation. This can be achieved by defining a collection of points that models the target rigid body, which can be of interest in a variety of applications, such as navigation [15], collision detection [16], or vehicle path prediction [17]. To name a few examples of the radio-based RBL approach, which is the main subject of this article, is the method in [18], where a MDS based approach was proposed that estimates the rigid body parameters efficiently in scenarios with incomplete observations, unaware of the targets shape. Another example is described in [19], in which a two-stage approach was used to estimate rotation, translation, angular velocity and

translational velocity by range and Doppler measurements, making use of various weighted least square (WLS) minimization methods. Finally, in [20], the problem was solved via Gaussian belief propagation (GaBP), estimating not only the stationary parameters, but also the velocity of the target.

In view of the above, we propose a first SMDS-based approach for RBL. By jointly exploiting distance and angle measurements, the method first estimates target point locations using a modified SMDS algorithm, followed by the estimation of the rigid body’s translation and rotation.

The structure of the remainder of article is as follows. First, a description of the rigid body system model and the SMDS edge kernel is offered in Section II. Then, in Section III, the proposed method for the estimation of the target rigid bodies’ landmark points, translation, and orientation is introduced. Finally, a comparison of the proposed scheme with the MDS-based RBL method, as well as an SMDS approach that only utilizes distance measurements, and the corresponding CRLB is presented in Section IV.

II. RIGID BODY LOCALIZATION SYSTEM MODEL

A. Rigid Body System Model

Consider a collection N of target points $\mathbf{c}_n \in \mathbb{R}^{2 \times 1}$ in the two-dimensional (2D) space¹, with $n = \{1, \dots, N\}$ that represent a rigid body, as shown in Figure 1. The structure and shape of said rigid body is therefore described by the corresponding conformation matrix \mathbf{C} constructed by the column-wise collection of the vectors \mathbf{c}_n . Then, consider the location \mathbf{S} that the rigid body moved to from its original conformation, which can be modeled by the following relationship, as

$$\mathbf{S} = \mathbf{Q} \cdot \mathbf{C} + \mathbf{t} \cdot \mathbf{1}_N^T, \quad (1)$$

where $\mathbf{t} \in \mathbb{R}^{2 \times 1}$ is a translation vector given by the difference of the geometric centers of the body at the two locations, $\mathbf{1}_N$ is a column vector with N entries all equal to 1, and $\mathbf{Q} \in \mathbb{R}^{2 \times 2}$ is a rotation matrix² determined by corresponding angle α , namely

$$\mathbf{Q} \triangleq \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}. \quad (2)$$

In addition to the rigid body, there are M anchor nodes $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_m, \dots, \mathbf{a}_M] \in \mathbb{R}^{2 \times M}$ in the environment, where all nodes are capable of measuring distance and angle information to each other.

¹Due to the use of complex numbers, the approach is limited to 2D networks. The extension to 3D is trivial, but laborious and therefore omitted.

²Note that the rotation matrix is in the $SO(2)$ group, *i.e.*, $\mathbf{Q}^T \mathbf{Q} = \mathbf{I}$ and $\det(\mathbf{Q}) = 1$, which can easily be extended to 3D, by the yaw, pitch and roll angles.

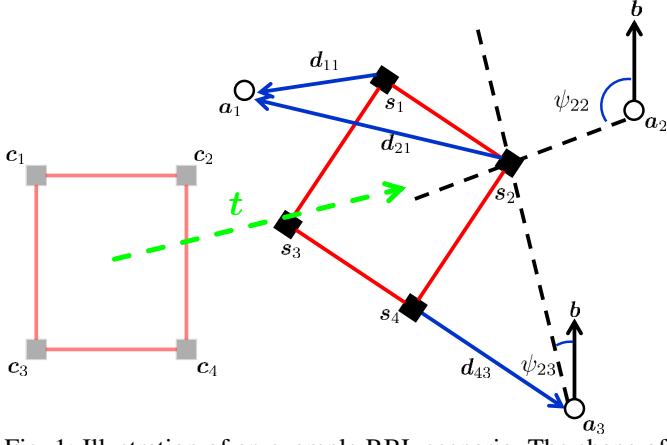


Fig. 1: Illustration of an example RBL scenario. The shape of the rigid body is defined by the distinct conformation matrices C . The translation vector t between the two locations is depicted in green. All nodes are able to perform distance and angle of arrival measurements to each other, where angles are measured w.r.t. the vector b .

B. SMDS Edge Kernel Construction

In the network, a total of $T = N + M$ nodes are placed in 2D space, where the anchor nodes locations are known, while the location of the rigid bodies landmark points are unknown. Following [21], the coordinates of the i -th node can be expressed as a complex number $x_i = a_{x_i} + jb_{x_i}$ such that the coordinates of all nodes can be collected in

$$\mathbf{X} \triangleq [\mathbf{A} \mid \mathbf{S}]^T \in \mathbb{C}^{T \times 2}. \quad (3)$$

Next, consider the set of unique ascending index pairs $\mathcal{P} \triangleq \{(1, 2), \dots, (1, T), (2, 3), \dots, (T-1, T)\}$, such that each element $p \in \mathcal{P}$ corresponding to a pair of indices (i, j) relates to the *complex edge* v_p , defined as

$$\begin{aligned} v_p &= (a_{x_j} - a_{x_i}) + j(b_{x_j} - b_{x_i}) \\ &= a_{v_p} + jb_{v_p} = d_p(\cos \theta_p + j \sin \theta_p), \end{aligned} \quad (4)$$

where d_p is the Euclidean distance between the pair of nodes which is given by

$$d_p \triangleq \|v_p\| = \|\mathbf{x}_i - \mathbf{x}_j\|, \quad (5)$$

where \mathbf{x}_i denotes the i -th row of the node matrix \mathbf{X} .

In total there are $P = \binom{T}{2} = T(T-1)/2$ complex edges, which, through the ascending index pairs, can be grouped as

$$\mathbf{v} = [\mathbf{v}_{AA}^T \mid \mathbf{v}_{AT}^T \mid \mathbf{v}_{TT}^T]^T = \mathbf{C} \cdot \mathbf{x} \in \mathbb{C}^M, \quad (6)$$

where \mathbf{v}_{AA} represents the anchor-to-anchor, \mathbf{v}_{AT} the anchor-to-target and \mathbf{v}_{TT} the target-to-target complex edges and the coefficient matrix is given by

$$\mathbf{C} \triangleq [\mathbf{C}_{AA}^T \mid \mathbf{C}_{AT}^T \mid \mathbf{C}_{TT}^T]^T, \quad (7)$$

with

$$\mathbf{C}_{AA} \triangleq \begin{bmatrix} \mathbf{1}_{M-1 \times 1} & & -\mathbf{I}_{M-1 \times M-1} & \mathbf{0}_{M-1 \times N} \\ \mathbf{0}_{M-2 \times 1} & \mathbf{1}_{M-2 \times 1} & -\mathbf{I}_{M-1 \times M-1} & \mathbf{0}_{M-2 \times N} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{1 \times M-2} & & \mathbf{1} & -\mathbf{1} & \mathbf{0}_{1 \times N} \end{bmatrix}, \quad (8a)$$

$$\mathbf{C}_{AT} \triangleq \begin{bmatrix} \mathbf{1}_{N \times 1} & & \mathbf{0}_{N \times M-1} & -\mathbf{I}_{N \times N} \\ \mathbf{0}_{N \times 1} & \mathbf{1}_{N \times 1} & \mathbf{0}_{N \times M-2} & -\mathbf{I}_{N \times N} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{N \times M-1} & & \mathbf{1}_{N \times 1} & -\mathbf{I}_{N \times N} \end{bmatrix}, \quad (8b)$$

$$\mathbf{C}_{TT} \triangleq \begin{bmatrix} \mathbf{0}_{N-1 \times M} & \mathbf{1}_{N-1 \times 1} & & -\mathbf{I}_{N-1 \times N-1} \\ \mathbf{0}_{N-2 \times M} & \mathbf{0}_{N-2 \times 1} & \mathbf{1}_{N-2 \times 1} & -\mathbf{I}_{N-1 \times N-1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{1 \times M} & & \mathbf{0}_{1 \times N-2} & \mathbf{1} & -\mathbf{1} \end{bmatrix}. \quad (8c)$$

To construct the kernel needed for the SMDS reconstruction, the dissimilarity measure between two edges v_m and v_p can be defined by the complex product

$$\begin{aligned} \kappa_{mp} &= v_m^* v_p = d_m(\cos \theta_m - j \sin \theta_m) \cdot d_p(\cos \theta_p + j \sin \theta_p) \\ &= d_m d_p (\cos(\theta_p - \theta_m) - j \sin(\theta_p - \theta_m)) \\ &= d_m d_p (\cos \theta_{mp} - j \sin \theta_{mp}). \end{aligned} \quad (9)$$

From the dissimilarities of all pairs, the complex kernel matrix can be constructed as

$$\mathcal{K} = \mathbf{v}^* \mathbf{v}^T = \begin{bmatrix} \mathbf{v}_{AA}^* \mathbf{v}_{AA}^T & \mathbf{v}_{AA}^* \mathbf{v}_{AT}^T & \mathbf{v}_{AA}^* \mathbf{v}_{TT}^T \\ \mathbf{v}_{AT}^* \mathbf{v}_{AA}^T & \mathbf{v}_{AT}^* \mathbf{v}_{AT}^T & \mathbf{v}_{AT}^* \mathbf{v}_{TT}^T \\ \mathbf{v}_{TT}^* \mathbf{v}_{AA}^T & \mathbf{v}_{TT}^* \mathbf{v}_{AT}^T & \mathbf{v}_{TT}^* \mathbf{v}_{TT}^T \end{bmatrix} = \begin{bmatrix} \mathcal{K}_A & \mathcal{K}_1 & \mathcal{K}_2 \\ \mathcal{K}_1^* & \mathcal{K}_3 & \mathcal{K}_4 \\ \mathcal{K}_2^* & \mathcal{K}_4^* & \mathcal{K}_T \end{bmatrix}. \quad (10)$$

As shown in the state-of-the-art (SotA), the kernel \mathcal{K} is of rank one, such that the complex edge vector \mathbf{v} can be estimated through a low rank truncation method, as used in the MDS and SMDS algorithms [7], [9], namely

$$\hat{\mathbf{v}} = \sqrt{\lambda} \mathbf{u}, \quad (11)$$

where (λ, \mathbf{u}) is the largest eigenpair of \mathcal{K} .

Finally, when the complete edge vector estimate $\hat{\mathbf{v}}$ is available, the corresponding coordinate vector estimate $\hat{\mathbf{x}}$ can be obtained by the inversion of (6), written as

$$\hat{\mathbf{x}} = \mathbf{C}^{-1} \cdot \hat{\mathbf{v}}. \quad (12)$$

III. PROPOSED METHOD

In light of the above, the proposed method consists of two steps. The first step offers a variation of the SMDS framework, adapted to the rigid body scenario, which yields the estimated landmark point positions of the rigid body $\hat{\mathbf{S}}$, while the second step makes use of a standard least square minimization approach to estimate the translation t and rotation Q of the rigid body, given $\hat{\mathbf{S}}$.

A. Rigid Body Estimation

Inspired by the Turbo MRC SMDS algorithm [21], which improves the classical SMDS [7] for independent target nodes by taking into account only a minor of the complex edge kernel, the same concept can be applied to the scenario of a rigid body, where not only the anchor-to-anchor measurements are known, but also the target-to-target measurements. Thus, for the RBL variation, a different minor of the complex edge

kernel needs to be selected that takes into account the known information, which yields

$$\begin{bmatrix} \mathcal{K}_1 \\ \mathcal{K}_3 \\ \mathcal{K}_4^\top \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{AA}^* \\ \mathbf{v}_{AT}^* \\ \mathbf{v}_{TT}^* \end{bmatrix} \cdot [\mathbf{v}_{AT}^\top], \quad (13)$$

which can be rearranged to solve for \mathbf{v}_{AT} , as

$$\hat{\mathbf{v}}_{AT}^{(n+1)} = \frac{[\mathcal{K}_1^\top \quad \mathcal{K}_3^\top \quad \mathcal{K}_4]}{\|[\mathbf{v}_{AA} \quad \hat{\mathbf{v}}_{AT}^{(n)} \quad \mathbf{v}_{TT}]^\top\|^2} \begin{bmatrix} \mathbf{v}_{AA} \\ \hat{\mathbf{v}}_{AT}^{(n)} \\ \mathbf{v}_{TT} \end{bmatrix}. \quad (14)$$

Moreover, an initial estimate $\mathbf{v}_{AT}^{(0)}$ can easily be obtained by

$$\hat{\mathbf{v}}_{AT} = \frac{[\mathcal{K}_1^\top \quad \mathcal{K}_4]}{\|[\mathbf{v}_{AA} \quad \mathbf{v}_{TT}]^\top\|^2} \begin{bmatrix} \mathbf{v}_{AA} \\ \mathbf{v}_{TT} \end{bmatrix}. \quad (15)$$

The algorithm of equation (14) exploits *all* the extrinsic information contained in \mathcal{K} , albeit by using only a *smaller* portion of the latter. Finally, as shown in [21], after estimating the vector $\hat{\mathbf{v}}_{AT}$, equation (12) can be used to reconstruct the rigid body landmark points $\hat{\mathbf{S}}$.

While the algorithm and the construction of the complex kernel matrix relies on the measurements of the distance and angle information, in some scenarios it might only be possible to collect the distance measurements. In such circumstances, it is possible to estimate the angle information prior to the SMDS algorithm such that first, a MDS step is performed, followed by the reconstruction of the angles, which can then be used to construct the kernel.

B. Rigid Body Parameter Estimation

With the estimate of the rigid bodies landmark points in hand, a least squares approach can be applied to estimate the rotation and translation parameters, as proposed in [19]. The estimation of the rotation and translation from the relationship between two sets of points, *i.e.*, the transformed landmark point position $\hat{\mathbf{s}}_i$ and the original landmark point position \mathbf{c}_i , defined by the conformation matrix, is a well known problem in the SotA [22], [23] and can be written as a minimization problem, given by

$$\arg \min_{\mathbf{Q}, \mathbf{t}} \sum_{i=1}^N (\hat{\mathbf{s}}_i - (\mathbf{Q}\mathbf{c}_i + \mathbf{t}))^\top \mathbf{W}_i (\hat{\mathbf{s}}_i - (\mathbf{Q}\mathbf{c}_i + \mathbf{t})), \quad (16)$$

s.t. $\mathbf{Q} \in SO(K)$,

where \mathbf{W}_i is the weighing matrix, defined by the inverse of the covariance of $\hat{\mathbf{s}}_i$. To generalize the problem it can be rewritten by a non-negative scalar weighting, as

$$\arg \min_{\mathbf{Q}, \mathbf{t}} G = \sum_{i=1}^N w_i \|\hat{\mathbf{s}}_i - (\mathbf{Q}\mathbf{c}_i + \mathbf{t})\|^2, \quad (17)$$

s.t. $\mathbf{Q} \in SO(K)$,

with the weighting matrix given by $\mathbf{W}_i = w_i \mathbf{I}$.

Next, the weighted average values are given by

$$\bar{\mathbf{s}} = \sum_{i=1}^N w_i \hat{\mathbf{s}}_i / \sum_{i=1}^N w_i, \quad (18a)$$

$$\bar{\mathbf{c}} = \sum_{i=1}^N w_i \mathbf{c}_i / \sum_{i=1}^N w_i. \quad (18b)$$

To find a closed form solution, the first step is to set the derivative of G with respect to \mathbf{t} to zero, which yields

$$\mathbf{t} = \bar{\mathbf{s}} - \mathbf{Q}\bar{\mathbf{c}}. \quad (19)$$

Plugging (19) back into the objective function G and substituting $\tilde{\mathbf{s}}_i = \hat{\mathbf{s}}_i - \bar{\mathbf{s}}$ and $\tilde{\mathbf{c}}_i = \mathbf{c}_i - \bar{\mathbf{c}}$, G can be rewritten as

$$\begin{aligned} G &= \sum_{i=1}^N w_i \|\tilde{\mathbf{s}}_i - \mathbf{Q}\tilde{\mathbf{c}}_i\|^2 \\ &= -2 \sum_{i=1}^N w_i \tilde{\mathbf{s}}_i^\top \mathbf{Q}\tilde{\mathbf{c}}_i + \mathcal{P} \\ &= -2 \text{trace}(\mathbf{Q} \sum_{i=1}^N w_i \tilde{\mathbf{c}}_i \tilde{\mathbf{s}}_i^\top) + \mathcal{P}, \end{aligned} \quad (20)$$

where $\mathcal{P} = \sum_{i=1}^N w_i (\|\tilde{\mathbf{s}}_i\|^2 + \|\tilde{\mathbf{c}}_i\|^2)$ is an independent constant. Since minimizing G is equivalent to maximizing the trace of $(\mathbf{Q} \sum_{i=1}^N w_i \tilde{\mathbf{c}}_i \tilde{\mathbf{s}}_i^\top)$, the optimal solution [23] is for the rotation matrix is found by

$$\mathbf{Q} = \mathbf{V} \text{diag}([\mathbf{1}_{K-1}^\top, \det(\mathbf{V}\mathbf{U}^\top)]^\top) \mathbf{U}^\top, \quad (21)$$

where the singular value decomposition (SVD) of $\sum_{i=1}^N w_i \tilde{\mathbf{c}}_i \tilde{\mathbf{s}}_i^\top$ defines $\mathbf{U}\Sigma\mathbf{V}^\top$. Finally, plugging (21) back into (19) gives the solution for the translation vector.

IV. PERFORMANCE EVALUATION

In this section we provide numerical results that illustrate the performance of the proposed SMDS-based RBL method. The region of interest consists of a 10m-by-10m room equipped with 8 anchor nodes, and a rigid body with 8 landmark points, similar to the simplified version illustrated in Figure 1.

The corresponding distance measurements are modeled as gamma-distributed random variables [24] with the mean given by the true distance and a standard deviation σ . In turn, the AoA measurement errors are Tikhonov-distributed [8], with the concentration parameter $\rho \geq 0$ inversely proportional to the angular error variance. Due to the non-linear relationship between ρ and the angular error variances, the influence of angular errors is captured by the quantity ζ_θ , defined as the bounding angle of the 90th centered percentile, *i.e.*,

$$\zeta_\theta = \theta_B \left| \int_{-\theta_B}^{\theta_B} p_\theta(t; \rho) dt \right| = 0.9, \quad (22)$$

where $p_\theta(t; \rho)$ denotes the central Tikhonov distribution [8].

The estimation errors are denoted by ε and measured by the MSE of the difference between the estimates and true values of the target parameters, *i.e.*,

$$\varepsilon = \frac{1}{K} \sum_{k=1}^K |\hat{\mathbf{t}}^{(k)} - \mathbf{t}|_2^2, \quad (23)$$

for the translation vector, where $\hat{\mathbf{t}}^{(k)}$ denotes the estimate at a k -th realization, which can also be applied to the estimates of the rotation matrix, calculating the results by averaging $K = 10^3$ Monte-Carlo realizations.

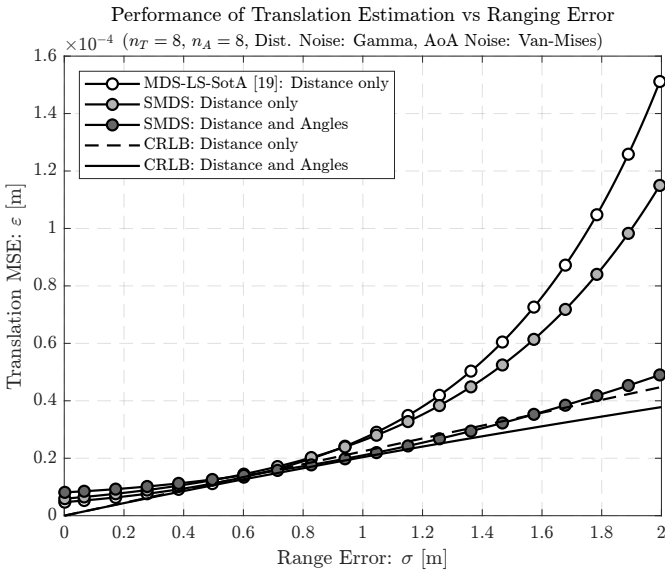


Fig. 2: MSE of the translation estimate of the proposed method and the SotA, over the range error σ .

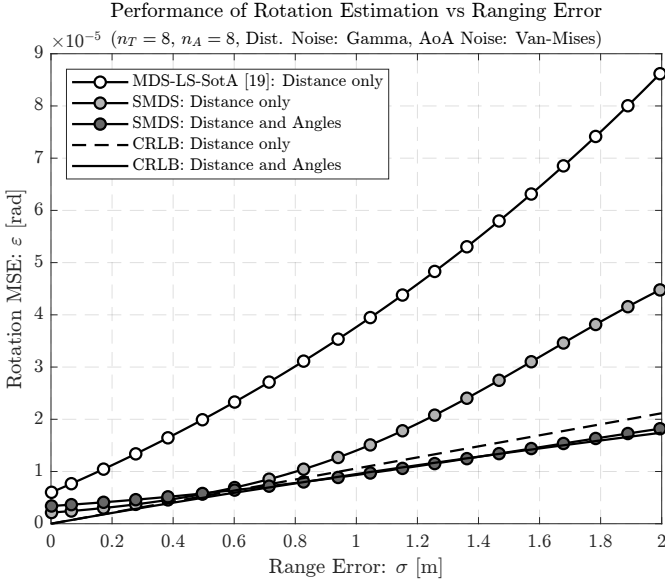


Fig. 3: MSE of the rotation estimate of the proposed method and the SotA, over the range error σ .

A. Fundamental Limits

To offer a detailed evaluation, the results of the proposed method are compared to the Cramér-Rao Lower Bound (CRLB), which offers a lower bound for the estimates. To that extend, [25] proposed a generalized framework for fundamental limits in rigid body localization, where the fisher information matrix (FIM) can be calculated in an information-centric approach knowing the pairs of measurements, their corresponding type, *i.e.*, distance or AoA measurements and the type of error the measurement is subject to.

Following [25], the FIM for the translation vector and the rotation matrix can be construed as

$$\mathbf{F}_t = \sum_{(n,a) \in \mathcal{P}_d} \lambda_{na} g'_t{}^d (g'_t{}^d)^\top + \sum_{(n,a) \in \mathcal{P}_\psi} \lambda_{na} g'_t{}^\psi (g'_t{}^\psi)^\top, \quad (24)$$

$$\mathbf{F}_Q = \sum_{(n,a) \in \mathcal{P}_d} \lambda_{na} g'_Q{}^d (g'_Q{}^d)^\top + \sum_{(n,a) \in \mathcal{P}_\psi} \lambda_{na} g'_Q{}^\psi (g'_Q{}^\psi)^\top, \quad (25)$$

where g' indicates the information gradient of the respective parameter and information type and λ denotes the information intensity defined by the type of error distribution, with the corresponding derivations for the specific parameters found in [25, Appendix A & B].

B. Numerical Results

Figure 2 and 3 show the result of the translation and rotation estimation for an MDS-based approach, an SMDS-based approach, where the angles are obtained by using only the distance measurements, and a full SMDS approach compared to the corresponding CRLBs. It can be observed that the distance only SMDS approach is performing better than the MDS approach, while not performing as good as the full SMDS approach, which is expected, since the full SMDS approach uses both distance and angle measurements.

In Figure 2 it can be observed that in general, the full SMDS yields the best results with estimates close to the CRLB, while the SMDS approach with only distance measurements is performing better than the MDS approach, and slightly better than the full SMDS in small range error regimes, which is expected since it was shown in [7] that the SMDS approach is not optimal in the small range error regime.

In Figure 3 similar properties can be observed, where the full SMDS approach is performing best, while the SMDS approach with only distance measurements is performing much better than the MDS approach, with the full SMDS approach performing close to the CRLB over the whole error range.

V. CONCLUSION

We proposed a novel SMDS-based RBL algorithm, which enables a rigid bodies relative translation (effective distance) and orientation (relative rotation) to be detected by a set anchor nodes the of another body, based only on a set of measurements of the distance and AoA information between sensors of the target and the anchor landmark points. A key point of the proposed method is that compared to conventional SMDS, the solution can be found in an iterative manner, while only a minor part of the complex edge kernel is considered that depends on the known noise-free measurements, which in the proposed scenario are anchor-to-anchor and target-to-target distances and angles. Simulation results illustrate the good performance of the proposed technique in terms of MSE as a function of the measurement error, reaching the fundamental limit illustrated by the CRLB. Future work will focus on the extension of the proposed method to 3D scenarios as well as the integration of further measurement types to enhance the estimation performance.

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