Binary Programming for Constellation-Aware Resource Allocation in Multiuser Networks

Yanwu Ding
Dept. Electrical and Computer Engineering
Wichita State University

Yirong Tu
Dept. Electrical and Computer Engineering
Wichita State University

***Abstract*—This paper addresses the challenge of efficient resource allocation in multiuser communication systems where quality-of-service (QoS) demands encompass both target data rates and bit error rate (BER) tolerance. We demonstrate that optimal resource allocation is fundamentally tied to modulation order selection for each user. We formulate integer optimization problems to maximize user capacity or minimize resource consumption (power, bandwidth, monetary cost) under heterogeneous user priorities and explicit QoS constraints. The solutions yield modulation-aware resource allocation policies that significantly improve system efficiency. Numerical results under both static and dynamic channel conditions validate the framework's effectiveness, showing adaptive performance gains in dynamic channel environments.**

Keywords*—*resource allocation, adaptive modulation, modulation order, QoS, constellation, integer optimization

I. Introduction

Optimal resource allocation is essential in multiuser systems to support high-data-rate applications while ensuring guaranteed quality of service (QoS), as established in prior work [1]–[6]. Common design objectives include minimizing power consumption, maximizing supported users, or improving signal-to-noise ratio (SNR), all subject to diverse QoS constraints [2]–[4], [6]–[8].

QoS requirements typically fall into two categories: the first specifies lower bounds on performance metrics such as channel capacity, data rate, throughput, or SNR; the second imposes upper bounds on data error rate, typically expressed as symbol error rate (SER), bit error rate (BER), or SNR. With fixed power and bandwidth, these categories often create conflicting constraints, leading many resource allocation strategies to address only one category.

Adaptive modulation is commonly used to maintain error rates below target thresholds [9]–[13]. For instance, [10] determines the maximum constellation size for M-PSK in single-user systems under minimum SNR requirements, while [11] jointly optimizes modulation order and transmit precoding in multiuser systems to minimize average SER under data rate constraints. Similarly, [12] and [13] explore modulation adaptation in NOMA and free-space optical systems, respectively.

In contrast to prior works that predominantly focus on a single QoS dimension, this paper proposes a unified resource allocation framework that simultaneously addresses both data rate and error tolerance requirements. By integrating adaptive modulation selection, our approach enables users to flexibly tailor their service requests, for instance, opting for lower data rates with stringent error tolerance for text-based applications, or higher data rates with relaxed error requirements for delay-sensitive services. Moreover, we explicitly formulate the resulting joint modulation-order selection and resource allocation problem as a constrained integer program, which accurately captures the discrete nature of modulation schemes and yields provably optimal allocation policies under heterogeneous user priorities.

Specifically, we formulate optimal resource allocation problems to maximize supported users while meeting QoS requirements within available bandwidth and power limits. The optimal solutions with appropriate modulation orders are obtained by solving constrained integer optimization problems. Higher-priority users are guaranteed QoS first, with remaining resources allocated to lower-priority users. While M-PSK and M-QAM modulations are used for illustration, the framework readily accommodates other modulation schemes.

II. Problem statement

We consider a multiple-access system where independent users communicate with a single base station receiver. Each user is equipped with a single antenna. The received signal from the j-th user is expressed as

$$r_j = \sqrt{P_{T_j}} h_j s_j + z_j, \tag{1}$$

where P_{T_j} denotes the transmit power, h_j is the channel coefficient, z_j is additive white Gaussian noise (AWGN) with zero mean and variance σ_j^2, and s_j is the transmitted symbol. The system employs either M-ary PSK (Phase Shift Keying) or quadrature amplitude modulation (QAM).

In this work, quality of service (QoS) for each user is characterized by two parameters: a target data rate R (in bps) and a maximum allowable bit error probability. The bandwidth required to achieve data rate R with M-ary modulation (PSK or QAM) is given by [14]: $B = \frac{2R}{M}$ Hz. Table I summarizes the required bandwidths for eleven different M-ary PSK and QAM modulations at data rates of $R = 10^6$ bps, 10^5 bps, and 10^4 bps, respectively.

TABLE I. M-PSK AND M-QAM MODULATIONS, BANDWIDTH, DATA RATES, AND SNR FOR $p_e = 10^{-5}$

Index	Modulation orders	M-ary	BW(KHz) $R = 10^6$ bps	BW(KHz) $R = 10^5$ bps	BW(KHz) $R = 10^4$ bps	SNR (dB) $p_e = 10^{-5}$	SNR (Value) $p_e = 10^{-5}$
1	2	BPSK	1000	100	10	6.9	4.9
2	4	QPSK	500	50	5	9.8	9.5
3	8	8PSK	250	25	2.5	15.2	33.1
4	16	16PSK	125	12.5	1.25	20.8	120.2
5	32	32PSK	52.5	6.25	0.625	26.6	457.1
6	64	64PSK	15.63	1.563	0.156	32.5	1778
7	8	8QAM	250	25	2.5	13.1	20.4
8	32	32QAM	52.5	6.25	0.625	19.5	89.1
9	64	64QAM	15.63	1.563	0.156	22.4	173.8
10	128	128QAM	7.81	0.781	0.078	25.4	346.7
11	256	256QAM	3.91	0.391	0.039	28.3	676.1

In addition to the target data rate, the QoS in the paper also requires the bit error probability to remain below a predefined threshold. For M-ary PSK modulation, the symbol error rate (SER) is given by [14], [15], $P_{\text{SER}}^{\text{PSK}} = 2Q\left(\sqrt{2\rho}\,\sin(\pi/M)\right)$, while for square M-ary QAM modulation ($M = L^2$), the SER is approximated by [16] $P_{\text{SER}}^{\text{QAM}} \approx 4Q\left(\sqrt{\frac{3\rho}{M-1}}\right)$, where $\rho = G|h|^2/\sigma^2$ is the instantaneous received signal-to-noise ratio (SNR), assumed constant over the channel coherence time, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-u^2/2)\,du$ is the Q-function. The user index j is omitted for notation simplicity. The corresponding bit error probability is approximated as $P_e \approx P_{\text{SER}}/\log_2(M)$ [16]. Figure 1 plots P_e versus SNR for both PSK and QAM modulations in an AWGN channel. Table I lists the required SNR values $\text{SNR}_{\text{value}} = 10^{\text{SNR}_{\text{dB}}/10}$ to achieve a target BER of $p_e = 10^{-5}$ for various PSK and QAM modulation schemes.

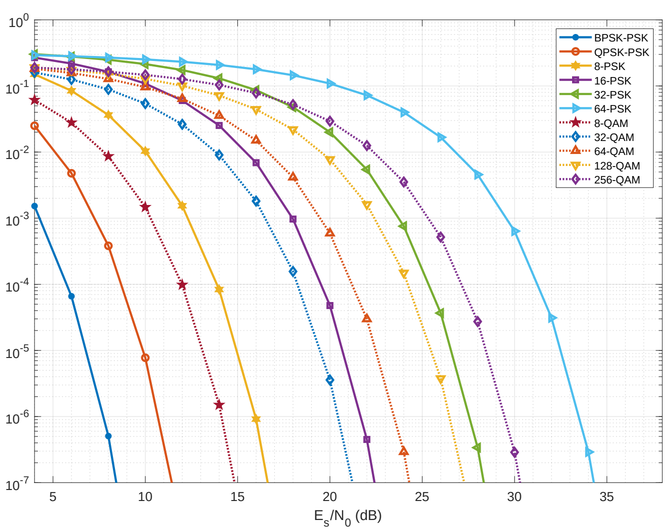

Fig. 1. BER for M-PSK M-QAM

III. JOINT RESOURCE ALLOCATION AND MODULATION SELECTION

A. Problem 1: Max number of users with Homogeneous Priorities subject to Power, Bandwidth, and QoS

This problem considers a simple case where all users have identical priority. The objective is to determine the maximum number of users that can be supported while satisfying the available power budget P_T, total bandwidth B_T, and individual QoS requirements on both data rate and bit error probability.

Let I represent the total number of available modulation orders. For example, Table I (column "Modulation Orders") shows $I = 11$ available modulation types. These are indexed from $i = 1$ to $i = 11$ in the first column of the table, corresponding to BPSK, 4-PSK, 8-PSK, ..., 64-PSK, 8-QAM, ..., and 256-QAM respectively.

1) Problem Formulation: Let the binary integer variable $x_{ij} \in \{0, 1\}$ indicate the assignment of a modulation order to a user, where $x_{ij} = 1$ indicates that the QoS requirement of the j-th user is satisfied using the i-th modulation order, while $x_{ij} = 0$ indicates that user j is not supported. Here, $i = 1, \ldots, I$ indexes the available modulation orders, and $j = 1, \ldots, J$ indexes users, with J representing a pre-defined upper bound on the number of supportable users. The problem of maximizing the number of users, subject to the total available power P_T, total bandwidth B_T, and individual QoS constraints, is formulated as the following constrained integer optimization problem:

$$\text{Problem 1:}\ \max \sum_j^J \sum_i^I x_{ij}(t) \tag{2a}$$

$$\text{s.t. } x_{ij}(t) = \{0, 1\} \tag{2b}$$

$$\sum_{j=1}^J \sum_{i=1}^I \left(\frac{\rho_i \sigma^2}{|h_j(t)|^2}\right) x_{ij}(t) \le P_T \tag{2c}$$

$$\sum_j^J \sum_i^I W_{ij}(t) x_{ij}(t) \le B_T \tag{2d}$$

$$P_{e_{ij}}(t) \le p_e, R_{ij}(t) \ge R, \forall i, j \tag{2e}$$

In (2e), $\rho_i(t)$ denotes the received SNR at time t required for modulation scheme i (from Table I) to achieve the maximum bit error probability P_e (e.g., 10^{-5}). Similarly, $W_{ij}(t)$ represents the bandwidth required at time t for the j-th user to achieve data rate R (e.g., 10^5 bps) using the i-th modulation order. We assume that all users have identical QoS requirements. Using Table I, an upper bound on the total number of users J is determined by considering the most demanding modulation orders in terms of power and bandwidth, respectively:

$$J = \min\left(\left\lfloor \frac{P_T}{P_{\max}} \right\rfloor, \left\lfloor \frac{B_T}{W_{\max}} \right\rfloor\right),$$

where $\lfloor\cdot\rfloor$ denotes the floor function, $P_{\max}$ is the maximum power required per user among all modulation orders, and $W_{\max}$ is the maximum bandwidth required per user.

Problem 1 is a binary linear programming problem. When all users experience an AWGN channel with unity channel gain and unit noise variance, they can be grouped according to their modulation order. Under these conditions, Problem 1 can be reformulated as the following integer optimization problem, denoted as Problem 1R, which can be solved efficiently using optimization solvers such as Gurobi [17]. In this case, since all users share identical channel conditions, multiuser diversity is absent; consequently, the solution to Problem 1R serves as a lower bound to that of Problem 1. Although this bound is conservative, it characterizes the worst-case scenario in practical dynamic system design.

$$\text{Problem 1R: } \max \ \mathbf{1}^T\mathbf{v} \tag{3a}$$

$$\text{s.t. } \ \mathbf{A}\mathbf{v} \preceq [P_T,\ B_T]^T \tag{3b}$$

$$\mathbf{v} \in \mathbb{Z}_+^{11} \tag{3c}$$

where $\mathbf{1}$ is an all-ones column vector whose dimension matches the size of $\mathbf{v}$ in (3a). To illustrate $\mathbf{v}$ and $\mathbf{A}$, we make use of Table I. The vector $\mathbf{v} = [v_1, v_2, \cdots, v_{11}]^T$ consists of non-negative integers, where v_i denotes the number of users transmitting with the i-th modulation order, $i = 1, \ldots, 11$. The objective of maximizing the total number of users is therefore expressed as $\max \sum_i v_i$ as in (3a). Matrix $\mathbf{A} = \begin{bmatrix} \mathbf{g} \\ \mathbf{b} \end{bmatrix}$ represents the required power (in watts) and bandwidth (in kHz) for the available modulation orders under the QoS constraint. For example, for $p_e = 10^{-5}$ and $R = 10^5$ bps, the vectors $\mathbf{g}$ and $\mathbf{b}$ obtained from Table I are as, $\mathbf{g} = [\ 4.9 \quad 9.5 \quad 33.1 \quad \cdots \quad 1778 \quad \cdots \quad 676.1\ ]$, and $\mathbf{b} = [\ 100 \quad 50 \quad 25 \quad \cdots \quad 1.563 \quad \cdots \quad 0.391\ ]$.

B. Problem 2: Multiuser Allocation with Heterogeneous Priorities under Resource and QoS Constraints

In contrast to Problem 1 where all users share identical priority levels, Problem 2 addresses scenarios with heterogeneous user priorities. Higher-priority users must have their Quality of Service (QoS) requirements satisfied first, with remaining power and bandwidth resources subsequently allocated to lower-priority users. Without loss of generality, we assume higher-priority users request higher data rates (R_1) compared to lower-priority users (R_2). For simplicity, all users share a common error tolerance threshold, though the proposed framework can be extended to accommodate multiple error tolerance levels.

The problem considers a two-tier priority structure comprising Level 1 (higher priority) and Level 2 (lower priority) users. Let N_1 denote the number of Level 1 users transmitting at rate R_1. The objective is to maximize the total number of supportable users while guaranteeing service for all N_1 Level 1 users, subject to total power budget P_T and bandwidth budget B_T. Residual resources after serving Level 1 users are allocated to Level 2 users transmitting at rate R_2. This sequential allocation process is formalized through a constrained integer optimization problem.

$$\text{Problem 2: } \max \sum_{i,j} x_{1,ij}(t) + \sum_{i,j} x_{2,ij}(t) \tag{4a}$$

$$\text{s.t. } \ x_{k,ij}(t) = \{0,1\}, k = 1,2 \tag{4b}$$

$$\sum_{i,j}\left(\left(\frac{\rho_{1,i}\sigma^2}{|h_{1,j}(t)|^2}\right)x_{1,ij}(t) + \left(\frac{\rho_{2,i}\sigma^2}{|h_{2,j}(t)|^2}\right)x_{2,ij}(t)\right) \le P_T \tag{4c}$$

$$\sum_{i,j}(W_{1,ij}(t)x_{1,ij}(t) + W_{2,ij}(t)x_{2,ij}(t)) \le B_T \tag{4d}$$

$$\sum_i \sum_j x_{1,ij}(t) \ge N_1 \tag{4e}$$

$$P_{e_{k,ij}}(t) \le P_e, R_{k,ij}(t) \ge R_k, \forall k, i, j \tag{4f}$$

where $x_{k,ij} = 1$ indicates that the j-th user at priority level k is served using the i-th modulation order with data rate R_k, where $k \in \{1,2\}$, $i = 1, \ldots, 11$, and $j = 1, \ldots, J$. The term $W_{k,ij}$ denotes the bandwidth required by user j when using modulation order i at data rate R_k, where R_1 and R_2 represent the data rates for priority levels 1 and 2, respectively. Similarly, a lower bound for the solution to Problem 2 can be obtained through a linear programming formulation where all users share the same channel gain.

IV. Numerical Results and Performance Analysis

To evaluate the resource allocation framework under dynamic channel conditions, and specify a lowered using the liearn programing. Rayleigh fading are applied for the channel realizations. The noise variance was normalized to unity for simplified power calculations. Unless stated otherwise, $P_T = 5000$ W, $B_T = 2000$ kHz, $p_e = 10^{-5}$, $R = 10^5$ bps, and the data in Table I is used in the simulations.

A. Problem 1

We first examine the lower bound on the maximum number of supported users. Solving Problem 1R yields a lower bound of 110 users, achieved with the following modulation assignment: 70 users employ 8-QAM, each requiring 20.4 W of power and 25 kHz of bandwidth, and 40 users employ 32-QAM, each requiring 89.1 W and 6.25 kHz. This allocation consumes 4992 W of power and 2000 kHz of bandwidth in total. It is noted that no PSK modulations are selected in this configuration. A separate simulation is then performed for only PSK modulations, the maximum number of supported users decreases to 91. In this case, 69 users use 8-PSK, each requiring 33.1 W and 25 kHz, and 22 users use 16-PSK, each requiring 120.2 W and 12.5 kHz. The resulting total resource consumption is 4928.3 W and 2000 kHz, leaving a slightly larger margin of unused power compared to the case where both PSK and QAM modulations are allowed.

o evaluate the performance of the proposed scheme for Problem 1 under dynamic channel conditions, simulations are conducted using Rayleigh fading channels over 500 independent runs. As shown in Fig. 2, the number of users supported varies across channel realizations, with a mean of 207.1 and a standard deviation of 3.1. This range highlights the algorithm's adaptability, maximizing capacity under favorable channel conditions while maintaining connectivity, albeit for fewer users, during deep fades.

Fig. 3 shows the selection frequency for each modulation, measured over the channel realizations. The results indicate a strong preference for specific orders, such as 8-QAM and 32-QAM, which offer the best trade-off between spectral efficiency and power cost under the prevailing QoS and channel constraints. This highlights the critical role of constellation-aware selection in optimizing system efficiency.

Fig. 4 confirms the efficient use of system resources, showing consistently high utilization rates for power and bandwidth over the first 200 trials. The system successfully adapts to channel variations without significant resource waste, maintaining stable and efficient operation.

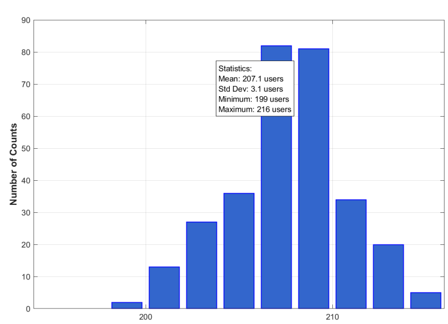

Fig. 2. Probelm1: Distribution of supported users across 100 channel realizations.

B. Problem 2

For Problem2, we set $P_T = 5000$ W, $B_T = 2000$ kHz, $N_1 = 30$, $p_e = 10^{-5}$, $R_1 = 10^5$ bps, and $R_2 = 10^4$ bps. considering Rayleigh fading, we conducted simulations with 500 channel realizations. The solution to Problem 2 demonstrates robust performance across channel variations.

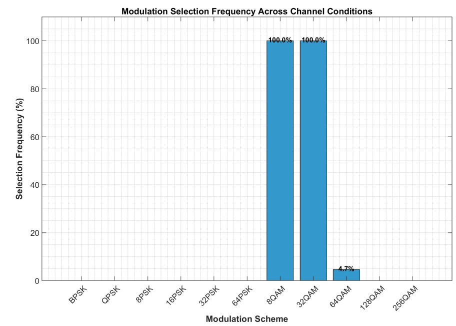

Fig. 3. Probelm1: Modulation selection frequency.

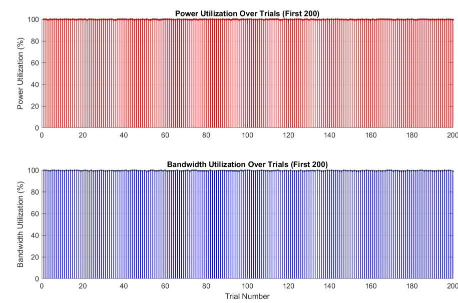

Fig. 4. Probelm1: Resource utilization over first 200 channel realizations.

Fig. 5 displays the distribution of supported users across Level 1 (high priority) and Level 2 (low priority). The allocation consistently satisfies the QoS constraints for both user levels, with Level 1 users guaranteed service while Level 2 users utilize remaining resources.

Fig. 6 shows the distribution of supported users, with a mean of 348 and standard deviation of 42. This result confirms that the framework successfully maintains Level 1 user priority while efficiently allocating residual resources to Level 2 users. The underlying modulation selection adapts to channel conditions to maximize total capacity while respecting QoS constraints.

Figure 7 shows the average computation time for solving Problem 2 across five resource sets: [1000W, 500kHz], [2000W, 1000kHz], [5000W, 2000kHz], [8000W, 3000kHz], and [10000W, 4000kHz]. The runtime analysis reveals the algorithm's scalability and computational efficiency under varying resource constraints.

V. Conclusion

This paper introduced a constellation-aware resource allocation framework for multiuser networks that jointly optimizes modulation order, power, and bandwidth allocation under quality-of-service constraints. We formulated and solved integer optimization problems for both homogeneous and heterogeneous user priority scenarios, establishing computable performance bounds for system design. The proposed approach demonstrates that adaptive modulation selection is essential for balancing spectral efficiency with power constraints, providing an effective methodology for resource management in constrained communication systems.

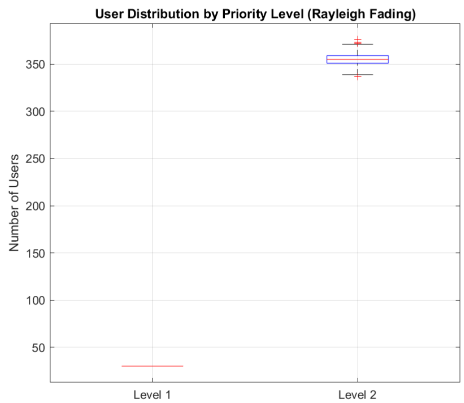

Fig. 5. Problem2: User priority distribution.

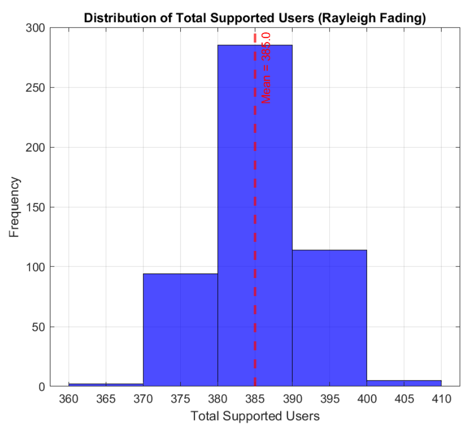

Fig. 6. Problem2: Distribution of total users distribution supported.

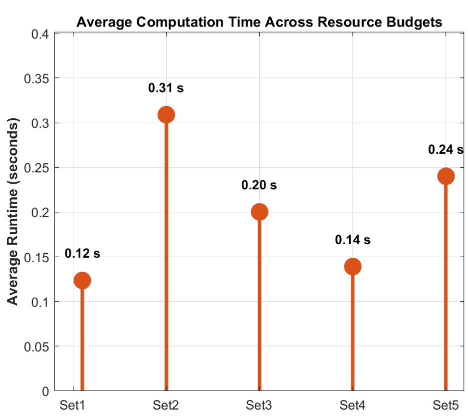

Fig. 7. Problem2: Average runtime for different sets of power and bandwidth

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