NextG Polar Code Design

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***Abstract*—Polar forward error correction (FEC) coded bits are transmitted via nonstationary(non-identical) orthogonal frequency division multiplexing (OFDM) subcarrier channels in a fifth generation (5G) network. Polar FEC and OFDM are also candidates for future G FEC and modulation. The signal-to-interference-plus-noise ratio (SINR) varies at each OFDM subcarrier channel and its estimation is and will be available in NextG. In this paper, it is proposed to permute OFDM subcarrier channel assignments for polar coded bit transmissions with the available SINR so that a smaller bit error rate (BER) be achieved. The results of the analysis and simulation simulation verify the benefits of the proposed polar code.**

I. Introduction

This paper addresses the question of optimizing a polar forward error correction (FEC) code design for nonstationary(non-identical) channels, specifically for use in 5G New Radio (NR) applications and NextG. Polarization of nonstationary channels was studied by Alsan and Telatar [1], who showed that polarization occurs under Arikan's polarization transform [2], and by Mahdavifar [3], who showed that a fast polarization can be achieved by a modified polar transformation process. Mahdavifar [3] also showed a two-stage polarization process that yielded polar coding schemes that approach the average capacity of the channels. The practical objective of this current paper is to consider the 5G wireless network setting where orthogonal frequency division multiplexing (OFDM) transmission is used, and to utilize the regularly generated signal-to-interference-plus- noise ratio (SINR) measurements for OFDM subchannels to improve polar code performance. The OFDM subcarrier channels are modeled as binary erasure channels with different erasure probabilities as done in [1]–[3]. In principle, the erasure probabilities, and hence the capacities of the polarized bit channels, can be computed recursively, but the complexity of this for a large number of channels is prohibitive (just as in the stationary case). The authors seek a low-complexity 5G-compatible approach to improve polar code performance by permuting the assignment of coded bits to OFDM subchannels based on knowledge of the subchannel SINRs and low complexity. Theoretical rigorous proofs are left for future study.

A. Motivation

In 5G networks, the polar FEC code in [2] has been used for control channels, for transmitting small packets of control information on both the uplink and the downlink, including the physical uplink control channel (PUCCH) and the physical downlink control channel (PDCCH) [4]. Also, in the 5G NR standards [4], it is required for a user equipment (UE) to report its SINR measured at the designated OFDM subcarrier reference signals (RSs) to the connected gNodeB aperiodically or periodically [4]. This report has been used for the channel quality indicator (CQI) in the OFDM subcarrier, synchronization, and for the allocation of resources block (RB). This SINR at the OFDM RSs can vary across the OFDM subcarrier channels. The polar encoded bits generated with the polar FEC coding are transmitted via OFDM subcarrier channels in the 5G networks. Can we utilize this prior knowledge of SINR also to improve polar FEC coding performance for future generation networks? This is the motivation of this paper.

B. Objective

The objective of this paper is to exploit the known SINR information and improve the bit error rate (BER) or block error rate (BLER) performance of a polar FEC coding.

C. Key Idea

The key idea in this paper is to rearrange OFDM subcarrier assignments for the coded bit transmissions to minimize the BER and to create a more consistent reliability sequence yielding better BER. This will be equivalent to creating an adaptive polar code reliability sequence depending on the SINR report. The same polar code Kronecker matrix structure in the conventional one [2] is used.

D. Proposed Method

In this paper, we use the available SINR at the specified RSs and estimate SINR for other OFDM subcarriers with simple interpolation. Hence, the sequence of SINR estimation is available at the TX. It is not necessary to know the channel coefficient itself at each OFDM subcarrier. Then, we find suboptimum OFDM subcarrier assignments for polar coded bit transmissions to create a consistent reliability sequence for coded and frozen bit selection to reduce BER.

E. Benefits

The following benefits can be achieved with the proposed simple polar code:

- The proposed system is compatible with existing ones because the same polar encoding structure with the Kronecker matrix in the existing 5G polar code system is used.
- The 5G SINR report is already available at both the UE and the gNodeB. Hence, no significant additional complexity is required. Only a simple search of a suboptimum

This work was supported in part by National Science Foundation (NSF) under Grants 2226447, 2326898, and 2515378.

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permutation index and the transmission of the searched permutation index written in $log_2(N = 2^l) = l$ number of bits at each codeword header are necessary.

- The proposed polar code under a nonstationary channel environment with simple OFDM subcarrier permutation can show a lower BER than a conventional polar code designed without permutation due to a more consistent reliability sequence.

II. Proposed System

A polar codeword is generated as a linear block code of length N as the product of an input message block of N bits times an $N \times N$ generator matrix G. For example, for $N = 4$, $(X_0, X_1, X_2, X_3) = (U_0, U_1, U_2, U_3)\ G^{(2)}$ where (X_0, X_1, X_2, X_3) and (U_0, U_1, U_2, U_3) are, respectively, the codeword and the message word, and $G^{(l)} = G_{2^l \times 2^l} = \begin{bmatrix} G^{(l-1)} & 0 \\ G^{(l-1)} & G^{(l-1)} \end{bmatrix}$ is the l-fold Kronecker matrix, e.g., $G_{2^0 \times 2^0} = 1$ and $G_{2 \times 2} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. For $N = 4$ in Fig. 1, each polar coded bit $X_0 = U_0 \oplus U_1$, $X_1 = U_1 \oplus U_3$, $X_2 = U_2 \oplus U_3$, or $X_3 = U_3$ has been transmitted through a binary-input discrete memoryless channel (BI-DMC) denoted by $W(\epsilon)$ when the component channel is stationary (i.e., identical) where $\oplus$ is the modulo 2 addition. In this paper binary erasure channel (BEC) is used to model the BI-DMC as done in [1]–[3]. Other channels, e.g., Rayleigh fading, will be studied in future. Similar BER or BLER improvement in this paper is expected for the other channels.

In practice, each component channel can be a nonstationary (i.e., the quality of the channels can be non identical or progressively changing) BEC of $W(\epsilon_0)$, $W(\epsilon_1)$, $W(\epsilon_2)$, and $W(\epsilon_3)$ as shown in Fig. 2 for $N = 4$. This is because the SINRs of OFDM subcarrier channels can vary across the subcarrier frequency as well as the time in practice, and an OFDM subcarrier channel can be modeled as an independent BEC as done in [1]–[3]. However, the BEC channels are not identical in this paper. The codeword size $N = 2^l$ is equal to the number of OFDM subcarrier channels. In addition, it is assumed that ϵ_i is known to the encoder and deterministic during a codeword transmission because $SINR_i$ is known, $i = 0, ..., N-1$. We can relate $SINR_i$ to ϵ_i. Approximately, ϵ_i is inversely proportional to $SINR_i$ because ϵ_i represents the coded bit erasure probability from a transmitted coded bit to the received coded bit through the ith independent OFDM subcarrier channel. In practice, we use an estimation of $SINR_i$. The estimation error will be modeled as a Mark process in future analysis.

In this paper, for example, it is proposed to transmit the coded bits X_0, X_1, X_2, and X_3 via the OFDM subcarrier channels with erasure probabilities $W(\epsilon_0)$, $W(\epsilon_2)$, $W(\epsilon_1)$, and $W(\epsilon_3)$, respectively, using a simple permutation matrix Π, as shown in Fig. 2 when $\epsilon_0 = \epsilon_1 = \epsilon$ and $\epsilon_2 = \epsilon_3 = 0.01\epsilon$, where ϵ represents the corresponding stationary BEC erasure probability from 0 to 0.5. This example corresponds to the case of $SINR_0 = SINR_1 \leq SINR_2 = SINR_3$. The permutation in Fig. 2 can be a suboptimal or optimal transmission strategy depending SINR conditions.

The existing polar code does not exploit the OFDM subcarrier SINR reports, i.e., no permutation, and may transmit the coded bits X_0, X_1, X_2, and X_3 via the OFDM subcarrier channels with erasure probabilities $W(\epsilon_0)$, $W(\epsilon_1)$, $W(\epsilon_2)$, and $W(\epsilon_3)$, respectively, as shown in Fig. 1. Then, this may yield a worse BER. On the other hand, the proposed polar code chooses OFDM subcarrier channels adaptively in block-by-block for which the BLER (i.e., the codeword error rate) is sufficiently small enough. The goal of this paper is to find a permutation which can yield the BLER close to the minimum BLER Π^* within a bound δ as

$$\hat{\Pi} = \text{ any of } \{\Pi : |BLER(\Pi) - BLER(\Pi^*)| \leq \delta\} \quad (1)$$

for a given set of $\epsilon_i, i = 0, ..., (N-1)$. The purpose of this proposed permutation is twofold. First, the reliability sequence in polar code, even under stationary channels, is very much not predictable. Under nonstationary conditions, the uncertainty of the reliability sequence increases all the more. Hence, it first serves the purpose of creating a consistent reliability sequence within the system. Second, the allocation of channel capacities is subject to change, depending on the permutation of the channels. The permutation we choose maximizes the $U2$ free bit channel capacity for the $N = 4$ case. We will propose a simple permutation that will make finding the reliability sequence simple, along with allocating most of the capacity to half the channels. Further rigorous theoretical analysis is left for future study.

III. Analysis

In a polar code, the "reliability sequence" refers to a list of bit positions ordered based on their reliability, meaning a sequence indicating which bits in the codeword are considered more reliable than others, used to determine which bits are information bits and which bits are frozen bits during encoding; it is essentially a ranking of the bit channels based on their transmission quality, e.g., channel capacity.

A. Channel Capacity for Nonstationary Binary Erasure Channel

The equivalent erasure probability for the $N = 2$ negatively (i.e., $s_0 = -$) or positively (i.e., $s_0 = +$) polarized channel in the nonstationary OFDM channels can be written as

$$f^{s_0} \begin{bmatrix} \epsilon_0 \\ \epsilon_1 \end{bmatrix} = \begin{bmatrix} \epsilon_0 + \epsilon_1 - \epsilon_0\epsilon_1 \text{ if } s_0 = - \\ \epsilon_0\epsilon_1 \text{ if } s_0 = + \end{bmatrix}. \quad (2)$$

Hence, the channel capacity, i.e., the reliability for the $N = 2$ nonstationary BEC channels can be written as

$$I(W^{s_0}) = 1 - f^{s_0} \begin{bmatrix} \epsilon_0 \\ \epsilon_1 \end{bmatrix}. \quad (3)$$

For $N = 2^l$, the equivalent erasure probability for the polarized channels in the nonstationary component channels can be written as

$$\epsilon_{W^{s_0, \cdots, s_{l-1}}(\epsilon_0, \ldots, \epsilon_{N-1})} =$$

$$f^{s_{l-1}}\begin{bmatrix} f^{s_{l-2}}\begin{bmatrix} \dots f^{s_2}\begin{bmatrix} f^{s_1}\begin{bmatrix} f^{s_0}\begin{bmatrix}\epsilon_0\\ \epsilon_{N/2}\end{bmatrix} \\ f^{s_0}\begin{bmatrix}\epsilon_{N/4}\\ \epsilon_{N/4+N/2}\end{bmatrix}\end{bmatrix} \\ f^{s_1}\begin{bmatrix} f^{s_0}\begin{bmatrix}\epsilon_{N/8}\\ \epsilon_{N/8+N/2}\end{bmatrix} \\ f^{s_0}\begin{bmatrix}\epsilon_{N/8+N/4}\\ \epsilon_{N/8+N/4+N/2}\end{bmatrix}\end{bmatrix}\end{bmatrix} \dots \\ \vdots \end{bmatrix} \\ f^{s_{l-2}}\begin{bmatrix} \vdots \\ \dots f^{s_2}\begin{bmatrix} f^{s_1}\begin{bmatrix} f^{s_0}\begin{bmatrix}\epsilon_{N-N/8-N/4-N/2-1}\\ \epsilon_{N-N/8-N/4-1}\end{bmatrix} \\ f^{s_0}\begin{bmatrix}\epsilon_{N-N/8-N/2-1}\\ \epsilon_{N-N/8-1}\end{bmatrix}\end{bmatrix} \\ f^{s_1}\begin{bmatrix} f^{s_0}\begin{bmatrix}\epsilon_{N-N/2-N/4-1}\\ \epsilon_{N-N/4-1}\end{bmatrix} \\ f^{s_0}\begin{bmatrix}\epsilon_{N-N/2-1}\\ \epsilon_{N-1}\end{bmatrix}\end{bmatrix}\end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{4}$$

Therefore, the corresponding polar code capacities for the nonstationary equivalent BEC channels of the $W^{s_0,\dots,s_{l-1}}(\epsilon_0,\dots,\epsilon_{N-1})$ transition probability can be written as

$$\begin{aligned} &I(W^{s_0,\dots,s_{l-1}}(\epsilon_0,\dots,\epsilon_{N-1})) \\ &= 1-\epsilon_{W^{s_0,\cdots,s_{l-1}}(\epsilon_0,\dots,\epsilon_{N-1})}. \end{aligned} \tag{5}$$

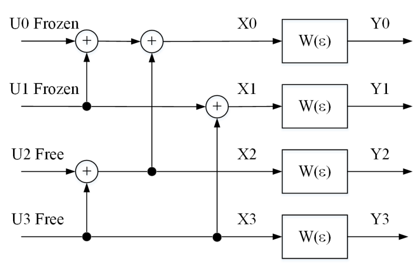

Figure 1: Stationary N = 4-vector BEC(ϵ).

Equation (4) is an equivalent erasure probability in a closed form for the $\{s_0,\dots,s_{l-1}\}$ polarized channels when the nonstationary erasure probability set $\{\epsilon_0,\dots,\epsilon_{N-1}\}$ is known to the transmitter during a codeword transmission period. This is computable in practice because the 5G SINR report is mandatory and the resource block assigned by a gNodeB is within a channel coherence interval [5]. This closed form is usable in the quick search of a suboptimum permutation in (1). It can also be exploitable for a brute force optimum search, but the computational complexity increases prohibitively due to a large number of possible permutations in $\{\epsilon_0,\dots,\epsilon_{N-1}\}$ as N increases.

For fair comparison between a stationary and a nonstationery BEC, we assume that the average erasure probability for the nonstationery BECs is equal to the stationary BEC erasure probability:

$$\epsilon = \frac{1}{N}\sum_{i=0}^{N-1}\epsilon_i, \quad 0 \le \epsilon_i \le 1. \tag{6}$$

Then, a polar code positively polarization capacity under nonstationary BEC channels is greater than or equal to a polar code positively polarization capacity under stationary BEC channels, i.e.,

$$\begin{aligned} &I_{iden}(W^{s_0=+,\dots,s_{l-1}=+}(\epsilon,\dots,\epsilon)) \\ &\le I_{noniden}(W^{s_0=+,\dots,s_{l-1}=+}(\epsilon_0,\dots,\epsilon_{N-1})). \end{aligned} \tag{7}$$

Equation (7) can be proved using the fact that a geometric mean is smaller than or equal to an arithmetic mean.

Also, it can be shown that the polar code negative polarization capacity under a nonstationary channel environment is lower than or equal to a polar code negative polarization capacity under a stationary channel environment, i.e.,

$$\begin{aligned} &I_{iden}(W^{s_0=-,\dots,s_{l-1}=-}(\epsilon,\dots,\epsilon)) \\ &\ge I_{noniden}(W^{s_0=-,\dots,s_{l-1}=-}(\epsilon_0,\dots,\epsilon_{N-1})). \end{aligned} \tag{8}$$

Therefore, a nonstationary vector channel can yield an improved polarization for a polar code than a stationary vector channel.

The bit error rate of a polar code with $N = 2$ codeword length and successive cancellation decoding can be approximated with log likelihood ratio of u_o under a BEC vector channel $(W(\epsilon_0), W(\epsilon_1))$ as

$$P_b(E) \simeq \epsilon_0\epsilon_1. \tag{9}$$

The BER is not changed for an $N = 2$ polar code when a coded bit X_1 is transmitted through a better or a worse channel, i.e., when a worse BEC of ϵ_0 channel and a better BEC channel of ϵ_1 are swapped. This is because the BER in (9) is symmetric in ϵ_0 and ϵ_1. However, this symmetry is broken when $N \ge 4$, depending on the combinations of better

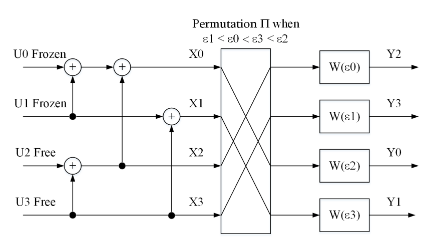

Figure 2: Proposed polar code for nonstationary N = 4-vector BEC W(ϵ_i) when $\epsilon_0 = \epsilon_1 = \epsilon, \epsilon_2 = \epsilon_3 = 0.01\epsilon$ as an example.

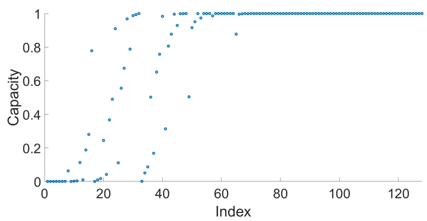

Figure 3: Polarized channel capacity for $N = 128$ nonstationary BEC at $\epsilon = .5$. Channels 1 through 64 = $.99\epsilon$, and channels 65 through 128 = $.01\epsilon$.

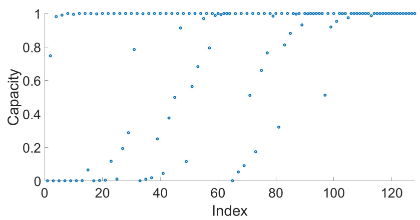

Figure 4: Polarized channel capacity for $N = 128$ nonstationary BEC at $\epsilon = .5$. Every other channel = $.99\epsilon$ and every other channel = $.01\epsilon$.

and worse BEC channels, e.g., the one in Fig. 2 is the best, while the one in Fig. 1 is the worst. If there is no permutation under a nonstationary BEC vector channel environment, then the conventional polar code as shown in Fig. 1 can perform the worst. This will be observed more in Section IV.

IV. Results and Discussion

The high N case is much more complicated, due to the symmetry found in (4), and optimization is very difficult to determine even for our narrowed requirements. However, due to the symmetry of all the negatively and positively polarized channels, i.e., $(s_0, \cdots, s_{l-1}) = (-, \cdots, -)$ and $(+, \cdots, +)$ in (4), we can apply the previous suboptimization strategy: For $N = 128$, select a good channel set for 1 through 64 coded bits and a bad channel set for 65 through 128 coded bits. This was found to create a simplistic capacity distribution for a half rate code in Fig. 3. On the other hand, if a good and a bad channel are selected alternatively for 1 through 128 coded bit transmissions, then this strategy becomes a very spread out distribution of polarized capacity, as shown in Fig. 4. These observations also suggest a new reliability pattern under permutation, and show the effect of permutation on reliability sequence.

Fig. 5 shows the $N = 4$ BER versus the erasure probability ϵ for both the proposed and the conventional worst-case polar codes under the nonstationary BEC channels of erasure probability $\epsilon_0 = \epsilon_1 = .99\epsilon$ and $\epsilon_2 = \epsilon_3 = 0.01\epsilon$. The optimization was the result of the capacity analysis. The proposed polar code BER in blue color shows about two decades smaller BER than the conventional worst-case BERs

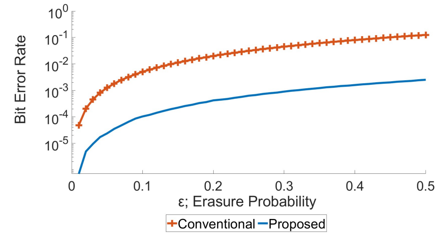

Figure 5: Polarized channel BER for $N = 4$ nonstationary BECs used in conventional and proposed polar codes.

in red color for all ϵ. This is because the proposed polar code, as shown in Fig. 2, transmits the coded bits X_1 and X_2 via $W(\epsilon_2)$ and $W(\epsilon_1)$, respectively, with a simple swapping. However, the conventional worst-case polar code, as shown in Fig. 1, transmits the coded bits X_1 and X_2 via $W(\epsilon_1)$ and $W(\epsilon_2)$, respectively, without subcarrier permutation. A simple permutation between $W(\epsilon_1)$ and $W(\epsilon_2)$ can yield about two decades BER improvement over the conventional worst-case polar code for a given nonstationary vector channel of $\epsilon_0 = \epsilon_1 = .99\epsilon$ and $\epsilon_2 = \epsilon_3 = 0.01\epsilon$.

Fig. 6 shows the BER versus the erasure probability ϵ for the $N = 128$ BEC vector channel case with the proposed scheme and the conventional one. For the proposed scheme, (4), (5), and a suboptimal permutation Π resulting in a consistent reliability sequence and hence smaller BER or BLER that approaches the minimum BLER Π^* in (1) were used. The BER in red is for the conventional polar code with randomly permuted channel and the reliability sequence, i.e., permutation fixed as the original polar code bit transmission in [2] and the BER in blue is for the polar code bit transmission with the proposed permutation and reliability sequence obtained from (4) and (5), when $\epsilon_0 = \cdots = \epsilon_{63} = .99\epsilon$ and $\epsilon_{64} = \cdots = \epsilon_{127} = .01\epsilon$. Significant BER improvement over the conventional scheme can be achieved for the proposed method. This is possible because the conventional polar scheme uses randomly permuted channels yielding an inconsistent reliability sequence, while the proposed scheme uses permuted channels yielding a consistent reliability sequence. In 5G, SINR report is available and hence code block channel condition can be given before every codeword transmission.

V. Proposed System

The addition of a permutation adds complexity to the already complex issue of finding an optimum reliability sequence, hence, we propose a fixed permutation to create a consistent reliability sequence. In a low-polarization environment, the reliability sequence will be identical to the current standard 5G reliability sequence regardless of permutation, but in high-polarization scenarios, the permutation has a high effect on capacity allocation as shown in Fig. 3 and Fig. 4. We propose investigating an optimum permutation and reliability selection algorithm in the future, but for the scope of this work we will only examine the simple solution of fixing the permutation of channels using the channel quality information.

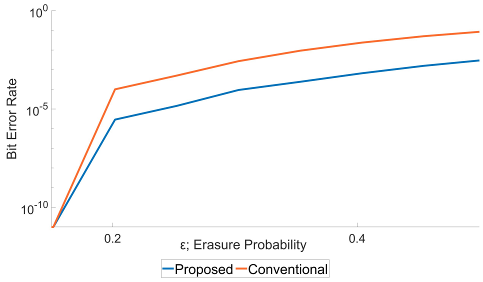

Figure 6: Polarized channel capacity for $N = 128$ nonstationary BECs used in conventional and proposed polar codes.

First, we determine the level of polarization that the channels are experiencing. We examined the sorted channels case, which is a simple permutation and can suboptimally allocate capacity, but more importantly, have shown that the reliability sequence is more consistent when the channels are permutated in a consistent manner. The reliability sequence shifts based on the level of polarization being experienced, hence we propose using the ratio between the average channel qualities of the best and worst half of channels to establish a polarization level, and then use this value to determine a reliability sequence which is predetermined at a given polarization level using (4). This solution allows for a suboptimal solution when brute forcing using (4) for all possible permutations is not feasible under the time and power constraints. Here is a summary of the steps we take for this method:

- A list of reliability sequences are predetermined based on the level of polarization the channels are experiencing. Using equation (4) we find these sequences under the following conditions. Where the capacity equations are written as follows: $I(W^{s_0,\cdots,s_{l-1}}(\epsilon_0,\ldots,\epsilon_{N-1})) = 1 - \epsilon_{W^{s_0,\cdots,s_{l-1}}(\epsilon_0,\ldots,\epsilon_{N-1})}$ The channel qualities is defined as follows:

$$\epsilon_{1,2,\cdots,N/2} = (1-P)\epsilon \tag{10}$$

$$\epsilon_{N/2+1,N/2+2,\cdots,N} = P\epsilon \tag{11}$$

Where ϵ can be selected to be a value between 0 and 1. ϵ should be selected to be a value around .5, because at high and low ϵ values the channel quality can behave erratically. We quantize the values of P and ϵ depending on memory constraints and generate a number of reliability sequences for various scenarios.

- We find the value of P and ϵ where $\epsilon_1 > \epsilon_2 > \cdots > \epsilon_N$ by averaging the quality of the best and worst half of channels such that:

$$2/N \sum_{n=1}^{N/2} \epsilon_n = (1-P)\epsilon \tag{12}$$

$$2/N \sum_{n=N/2+1}^{N} \epsilon_n = P\epsilon \tag{13}$$

which can be solved as:

$$P = \sum_{n=N/2+1}^{N} \epsilon_n / \sum_{n=1}^{N} \epsilon_n \tag{14}$$

$$\epsilon = 2/N \sum_{n=1}^{N} \epsilon_n \tag{15}$$

- Using this value, a reliability sequence is chosen from the list determined from the first step using the value P and the ϵ value determined. It should be noted that in Fig. 6 the proposed case doesn't use quantization, but uses an ϵ value of .5 and the P value is fixed as .99

In other words, reliability sequences should be pre-prepared using the provided non-identical capacity equations in (5) under a specified polarization value, then should be selected based on the polarization value. Using (4) a variety of reliability sequences can be determined under differing levels of polarization. This method is designed for the case that the encoder does not have the processing power to fully calculate the capacities of the channel creating a new reliability sequence each time. Fig. 6 shows the use of this proposed scheme where channels are permuted and a reliability sequence is selected based on polarization level, and the conventional where channels are not permuted.

VI. Conclusion

For the existing and 5G polar-coded systems, there has been not much discretion in the OFDM subcarrier carrier assignments for the coded bit transmissions. In practice, the 5G OFDM subcarriers can have nonistationary transition probabilities because the SINR varies, depending on the OFDM subcarrier index. In this paper, it was proposed to simply permute the OFDM subcarrier assignment and transmit the coded bits. The proposed polar-coded system is implementable and requires insignificant additional complexity because the same polar code Kronecker matrix is used and 5G SINR report is available at both the UE and gNodeB. Furthermore, in this paper the equivalent erasure probability and the corresponding mutual information, i.e., polarized channel capacity for a BEC vector channel of length $N = 2^l$ were derived when the component BEC channel is nonstationary. Finally, it was shown that a nonstationary vector channel has a higher positively polarization and a lower negatively polarization than an stationary channel.

References

[1] M. Aslan and E. Telatar, "A proof of polarization and polarization for non-stationary memoryless channels," *IEEE Transactions on Information Theory*, vol. 62, no. 9, pp. 4873–4878, 2016.

[2] E. Arıkan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Transactions on Information Theory*, vol. 55, no. 7, pp. 3051–3073, 2009.

[3] H. Mahdavifar, "Polar coding for non-stationary channels," *IEEE Transactions on Information Theory*, vol. 66, no. 11, pp. 6920–6938, 2020.

[4] 3GPP, "5G; NR; physical layer measurements," 3rd Generation Partnership Project (3GPP), Technical Specification TS 38.215 version 17.5.0, Release 17, March 2023.

[5] H. Y. Thomas Marzetta, Erik Larson and H. Ngo, "Fundamentals of massive mimo," in *Cambridge University Press*, 2016.