Performance Analysis of Cooperative RSMA-SWIPT Communication System

Jia qi Yan
School of Electronics & Information
Soochow University
Suzhou, China
uniyanjq@126.com

Jia Hou
School of Electronics & Information
Soochow University
Suzhou, China
houjia@suda.edu.cn

Abstract—In this paper, a cooperative RSMA simultaneous wireless information and power transfer (SWIPT) system is presented. Specifically, in the system, the near user is treated as a relay to realize the cooperative transmission for the far user by using the collected energy. Based on the network characteristics, two resource allocation rules are analyzed. A closed-form expression for the outage probability (OP) of the near user and a closed-form approximation expression for the OP of the far user are derived. In addition, the total throughput formula is derived based on the OP. Considering the user decoding performance and OP, to obtain the optimal total throughput, this study uses genetic algorithm to solve the total throughput optimization problem. The simulation results show that the RSMA-SWIPT system in this paper has more flexibility than compared with NOMA, the OP is significantly reduced, and the total throughput is improved by about 0.2bit/s/Hz.

Keywords—cooperative relaying, RSMA, SWIPT

I. INTRODUCTION

Rate-splitting multiple access (RSMA) technique has been considered as one improved non-orthogonal multiple access (NOMA) method[1]. It has been widely used in satellite communication [2] and unmanned aerial vehicles (UAVs) [3], with higher transmission efficiency. Otherwise, the simultaneous wireless information and power transfer (SWIPT) can achieve the better energy efficiency (EE) with higher information transmission rate [4],[8],[9].

RSMA was studied by many researchers due to its excellent performance. [1] proved that RSMA outperforms NOMA and SDMA in several scenarios. [5] demonstrated that the rate region of RSMA came much closer to the capacity region than SDMA and NOMA. It was discussed in [6] that with imperfect channel state information at the transmitter (CSIT), linear precoding RSMA could obtain a larger rate region than complex dirty paper coding (DPC). In [7], the RSMA cognitive radio system with SWIPT was discussed, and joint power allocation and power splitting (PS) were analyzed. In [8], a cooperative rate splitting (CRS) strategy was proposed based on a three-node relay channel communication model. In [9], RSMA was used for the multi-user (MU) SWIPT system; the weighted sum rate (WSR) was proposed to be maximized, constrained by the harvested energy and the maximum transmit power. However, it was not considered where the user could receive information and energy. In [10], a reconfigurable intelligent surface (RIS) assisted RSMA-SWIPT network was investigated, and it was proved that RSMA contributed significantly to the EE of the system compared to the conventional scheme.

However, most of the above papers focused on analyzing the EE and optimization of RMA-SWIPT. They did not consider the analysis of the network performance of cooperative RSMA-SWIPT system. Therefore, this paper proposes a cooperative two-user RSMA SWIPT system and analyzes its performance. The proposed model not only exploits the advantages of cooperative RSMA in network performance [5],[6] and user cooperation [7]-[9] but also exploits the advantages of SWIPT in [4].

The main contributions of this paper can be summarized as follows:

- Constructing a three-node communication model, and push to a closed expression for the outage probability (OP) of cell center users and an approximate closed expression for the OP of cell edge users. The derived analytical results are corroborated through Monte-Carlo simulations.
- The sum-throughput (bit/s/Hz) is investigated and analyzed optimally from the OP. A genetic algorithm is employed to optimize the power allocation factor to maximize the sum-throughput by continuously iterating the population size.
- Based on the principles of SWIPT relay transmission, the power required for near-user decoding is reduced, leading to a lower overall throughput compared to the RSMA system. Therefore, considering the balanced performance of the system, a new resource allocation strategy is investigated, where far users decode their private messages first. the proposed system can achieve explicit rate region improvement and the sum-throughput is slightly increased compared to the original system.

II. SYSTEM MODEL

Fig. 1 consider an RSMA-SWIPT three-node downlink cooperative network downlink model, . In this case, the base station (BS) S acts as a signal transmitting source and communicates with two single-antenna users, U_1 and U_2. U_1 belongs to the near-end user, while U_2 has a poorer signal reception quality, and belongs to the far-end user.

According to the RSMA principle of multiple antenna broadcast channel[11], the three wireless channel links of the system can be expressed as h_1, h_2, and h_3, and assume they are all Rayleigh-flat fading. Its channel gain is an exponential random variable with probability density function (PDF), $f_{|h_i|^2}(z) = 1/\lambda_i \cdot \exp(-z/\lambda_i)$, where λ_i denotes the mean of $|h_i|^2$. Furthermore, the average channel gain can be denoted as $\mathrm{E}\left[|h_i|^2\right] = \mathcal{L}/(d_i/d_0)^l$, where d_i denotes the distance (in meter) between two nodes, l denotes the path loss exponent, d_0

denotes the reference distance, and $\mathcal{L}$ denotes the average signal power attenuation at d_0 . Let n_{aX}, n_{cX} denote the additive white Gaussian noise (AWGN) at the receiving antenna and the downconverter at node X, respectively, and $n_{aX} \sim \mathcal{CN}\left(0,\sigma_{aX}^2\right)$, $n_{cX} \sim \mathcal{CN}\left(0,\sigma_{cX}^2\right)$, where $X \in \{1,2\}$.

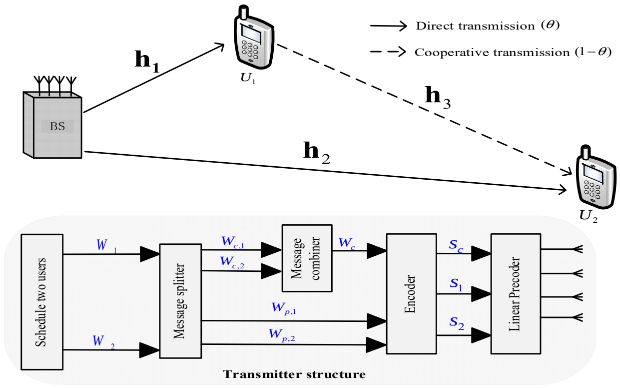

Fig. 1. An illustration of an RSMA-SWIPT transmission for a two-user system (Including RSMA transmitter structure).

Fig. 2 gives a schematic of the time allocation for the two transmission phases. Assume that the time used for one communication is T in the cooperative RSMA-SWIPT model, the proportion of time allocation is denoted by θ ,where $0 \le \theta \le 1$. The parameter ρ can allocate the power allocation of the cooperative U_1 . In stage I, the BS broadcasts a message, U_1 performing energy harvesting (EH) and information decoding (ID). In stage II, U_1 acts as a relay user, re-encoding the decoded public message to U_2 .

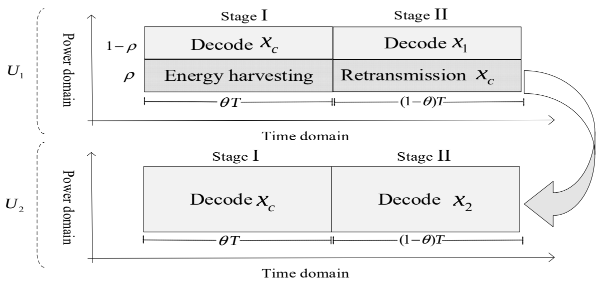

Fig. 2. Schematic diagram of the RSMA-SWIPT system.

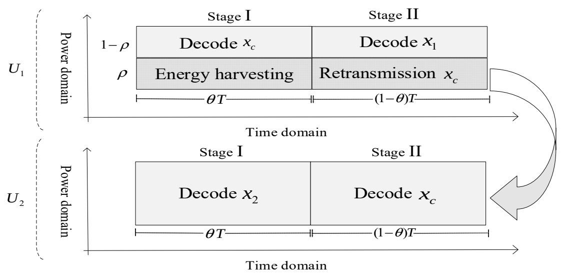

Fig. 3. Schematic diagram of the GF- RSMA-SWIPT system.

The transmission decoding process can be improved to obtain the strategy shown in Fig. 3. The system is assumed to be grant-free (GF) [12]. For U_2 , it decodes its private message first, decodes the public message in the second time slot and receives the message forwarded by U_1 for message merging. This transmission strategy is analyzed along with the original strategy and improves performance for U_2 user.

In summary, the collaborative RSMA-SWIPT system is carried out in two phases, these two phases correspond to the analysis in parts *A* and *B* below.

A. *The First Phase: EH and ID*

According to the RSMA principle, the message delivered to the two users is divided into three parts and merged into a vector $\mathbf{s}=[s_c,s_1,s_2]^T$ with a precoding matrix $\mathbf{w}=[w_c,w_{1,}w_2]$. Assume that $|h_1|^2>|h_2|^2$, the power allocation coefficients are $\{p_c,p_1,p_2\}$, where $p_c+p_1+p_2=1$ .The signal received by the is

$$\begin{aligned} y_k^1 &= h_k^H x + n_k \\ &= h_k^H w_c s_c + h_k^H w_1 s_1 + h_k^H w_2 s_2 + n_{ak} \end{aligned} \tag{1}$$

Using the PS based SWIPT protocol, the EH by U_1 can be expressed as: $E_1=\eta\rho P_S|h_1|^2\theta T$, where P_S is the base station transmit power and $P_S=P_1+P_2$; η denotes the energy conversion efficiency of the U_1 . For ID, the received signal at U_1 can be written as $y_1^{ID}=\sqrt{1-\rho}\left(h_1^H x n_{a1}\right)+n_{c1}$. The signal to-interference-plus-noise ratio (SINR) of its received signal can be expressed as

$$\xi_{c,1}=\frac{(1-\rho)p_cP_S|h_1|^2}{(1-\rho)\left(p_1P_S|h_1|^2+p_2P_S|h_1|^2+\sigma_{a1}^2\right)+\sigma_{c1}^2} \tag{2}$$

The SINR received at U_2 for decoding x_c can be written as

$$\xi_{c,2}=\frac{p_cP_S|h_2|^2}{\left(p_1P_S|h_2|^2+p_2P_S|h_2|^2+\sigma_{a2}^2\right)+\sigma_{c2}^2} \tag{3}$$

B. *The Second Phase: Cooperative Relay Transmission*

In phase II, users decode their own private signals individually and U_1 forwards the signals with energy E_1 . At this time, the signal received by U_2 is $y_2^2=h_3^H\sqrt{P_r}s_c+n_3$. The SINR of the user's decoded private signal is written as $\xi_{p,1},\xi_{p,2}$, and the SINR of the forwarded signal received by U_2 is written as ξ , respectively,

$$\xi_{p,1}=\frac{(1-\rho)p_1P_S|h_1|^2}{(1-\rho)\left(p_cP_S|h_1|^2+p_2P_S|h_1|^2+\sigma_{a1}^2\right)+\sigma_{c1}^2} \tag{4}$$

$$\xi_{p,2}=\frac{p_2P_S|h_2|^2}{\left(p_1P_S|h_2|^2+\sigma_{a2}^2\right)+\sigma_{c2}^2} \tag{5}$$

$$\xi=\frac{\rho\eta P_S|h_1|^2|h_3|^2}{\sigma_{a2}^2+\sigma_{c2}^2} \tag{6}$$

Next, the performance of the considered system model will be studied in terms of outage probability and sum-throughput in Sections III and IV, respectively.

III. Outage Performance Analysis

The OP of a user can be defined as the probability that the user's instantaneous data rate is lower than a predetermined target data rate [13]. Let $R_{th,c},R_{th,1},R_{th,2}$ (bit/s/Hz) denote the user's message $\{x_c, x_1, x_2\}$ target data rate , $a_1 \triangleq \frac{(1-\rho) p_c P_S}{(1-\rho)\sigma_{a1}^2 + \sigma_{c1}^2}$, $a_2 \triangleq \frac{(1-\rho) p_1 P_S}{(1-\rho)\sigma_{a1}^2 + \sigma_{c1}^2}$, $a_3 \triangleq \frac{(1-\rho) p_2 P_S}{(1-\rho)\sigma_{a1}^2 + \sigma_{c1}^2}$, $\mu_1 \triangleq \frac{a_1}{a_2 + a_3}$, $\mu_2 \triangleq \frac{a_2}{a_3}$, $b_1 \triangleq \frac{p_c P_S}{\sigma_{a2}^2 + \sigma_{c2}^2}$, $b_2 \triangleq \frac{p_1 P_S}{\sigma_{a2}^2 + \sigma_{c2}^2}$, $b_3 \triangleq \frac{p_2 P_S}{\sigma_{a2}^2 + \sigma_{c2}^2}$, $\tau_1 \triangleq \frac{\xi_1}{a_1 - (a_2 + a_3)\xi_1}$, $\tau_2 \triangleq \frac{\xi_2}{a_2 - a_3 \xi_2}$, $\vartheta_1 \triangleq \frac{b_1}{b_2 + b_3}$, $\vartheta_2 \triangleq \frac{b_3}{b_2}$, $\sigma_1 \triangleq \frac{\xi_3}{b_1 - (b_2 + b_3)\xi_3}$, $\sigma_2 \triangleq \frac{\xi_4}{b_3 - b_2 \xi_4}$. Parts A and B give the interruption probabilities for U_1 and U_2 , respectively.

A. *Outage Probability of U_1*

For U_1, an outage occurs at U_1 when the public message x_c cannot be decoded or when x_c is successfully decoded but x_1 is unsuccessful. Therefore, the OP of can be expressed as

$$P_{out}^N = \Pr\left(\xi_{c,1} < \xi_1\right) + \Pr\left(\xi_{c,1} \geq \xi_1, \xi_{p,1} < \xi_2\right) \tag{7}$$

where $\xi_1 = 2^{R_{th,c}} - 1$ and $\xi_2 = 2^{R_{th,1}} - 1$ denote the SINR thresholds used by U_1 to decode the message correctly.

Theorem 1: The closed expression for the OP of U_1 can be written as

$$P_{out}^N = \begin{cases} 1 - \exp(-\tau_1 / \lambda_1) & \xi_1 < \mu_1, \xi_2 < \mu_2, \tau_1 \geq \tau_2 \\ 1 - \exp(-\tau_2 / \lambda_1) & \xi_1 < \mu_1, \xi_2 < \mu_2, \tau_1 < \tau_2 \\ 1 & others \end{cases} \tag{8}$$

Proof: In (2), (4), and (7), the OP can be re-expressed as

$$\begin{aligned} P_{out}^N &= 1 - \Pr\left(\frac{a_1 X}{a_2 X + a_3 X + 1} \geq \xi_1, \frac{a_2 X}{a_3 X + 1} \geq \xi_2\right) \\ &= 1 - \Pr\left(X \geq \frac{\xi_1}{a_1 - (a_2 + a_3)\xi_1}, X \geq \frac{\xi_2}{a_2 - a_3 \xi_2}\right) \end{aligned} \tag{9}$$

It can be observed that if $\xi_1 \geq \mu_1$ or $\xi_2 \geq \mu_2$, $P_{out}^N = 1$; if $\xi_1 < \mu_1$, $\xi_2 < \mu_2$, by checking the relativity between τ_1, τ_2 and P_{out}^N , the expression for the closed form of the OP for U_1 , as shown in (8), can be obtained by some algebraic operations. This completes the proof of Theorem 1.

B. *Outage Probability of U_2*

As in Section II, while the OP of U_2 can be expressed as

$$P_{out}^F = 1 - \Pr\left(\max\left\{\xi_{c,2}, \xi\right\} \geq \xi_3, \xi_{p,2} \geq \xi_4\right) \tag{10}$$

where $\xi_3 = 2^{R_{th,c}} - 1$ and $\xi_4 = 2^{R_{th,2}} - 1$ denote SINR thresholds used by U_2 to decode messages x_c and x_2 , respectively.

Theorem 2: The closed approximate expression for the OP of can be written as

$$P_{out}^F = 1 - e^{-\frac{\sigma_k}{\lambda_1}} - \frac{2a}{\lambda_1 \lambda_3} \mathbb{K}_0\left(2\sqrt{\frac{a}{\lambda_1 \lambda_3}}\right) \times \left(e^{-\frac{\sigma_2}{\lambda_1}} - e^{-\frac{\sigma_1}{\lambda_1}}\right) \tag{11}$$

if $\sigma_1 \geq \sigma_2$, $k = 1$, otherwise, $k = 2$, if $\xi_3 \geq \vartheta_1$ or $\xi_4 \geq \vartheta_2$, $P_{out}^F = 1$ where $\mathbb{K}_0$ is a Type II modified Seibel function [14], where $\mathbb{K}_\nu(x) = \int_0^\infty \exp\left(-x\cosh(t)\right)\cosh(\nu t)dt$.

Proof: See Appendix 1

It can be further derived that the OP of U_2 in GF-RSMA-SWIPT system, changes with the following parameters: $\tilde{\xi}_{p,2} = \frac{p_2 P_S |h_2|^2}{\left(p_c P_S |h_2|^2 + p_1 P_S |h_2|^2 + \sigma_{a2}^2\right) + \sigma_{c2}^2}$, $\tilde{\sigma}_1 \triangleq \frac{\xi_3}{b_3 - (b_1 + b_2)\xi_3}$, $\tilde{\vartheta}_1 \triangleq \frac{b_3}{b_1 + b_2}$. At this point, the OP of U_2 is

$$\tilde{P}_{out}^F = \begin{cases} 1 - e^{-\frac{\sigma_1}{\lambda_1}} \times \frac{2a}{\lambda_1 \lambda_3} \mathbb{K}_0\left(2\sqrt{\frac{a}{\lambda_1 \lambda_3}}\right) & \tilde{\vartheta} < \zeta_3 \\ 1 & others \end{cases} \tag{12}$$

The proof is the same as Theorem 2.

IV. Optimal Sum-Throughput Analysis

In this section, the sum-throughput of the RSMA-SWIPT system is analyzed, denoted as $\mathcal{J}$. An optimization algorithm based on Genetic Algorithm [15] is proposed to find the optimal value of ρ , denoted as $\rho^{(*)}$, which maximizes the sum-throughput of the system. The sum-throughput of the system can be expressed as

$$\begin{aligned} \mathcal{J} &= \left(1 - P_{out}^N\right) R_{th,1} + \left(1 - P_{out}^N\right) R_{th,2} \\ &= R_{th,1} e^{-\frac{\varphi}{\lambda_1}} + R_{th,2} e^{-\frac{\gamma}{\lambda_1}} - \frac{2a}{\lambda_1 \lambda_3} \mathbb{K}_0\left(2\sqrt{\frac{a}{\lambda_1 \lambda_3}}\right) \times \left(e^{-\frac{\sigma_2}{\lambda_1}} - e^{-\frac{\sigma_1}{\lambda_1}}\right) \end{aligned} \tag{13}$$

if $\tau_1 \geq \tau_2$, $\varphi = \tau_1$ otherwise $\varphi = \tau_2$; similarly, if $\sigma_1 \geq \sigma_2$, $\gamma = \sigma_1$, otherwise $\gamma = \sigma_2$. For ease of calculation, let $\gamma = \sigma_1$, analyzing with $\tilde{P}_{out}^F$, the objective function $\mathcal{J}$ can be rewritten as

$$\mathcal{J} = R_{th,1} e^{-\frac{\varphi}{\lambda_1}} + R_{th,2} e^{-\frac{\sigma_1}{\lambda_1}} \times \frac{2a}{\lambda_1 \lambda_3} \mathbb{K}_0\left(2\sqrt{\frac{a}{\lambda_1 \lambda_3}}\right) \tag{14}$$

The problem under consideration is described as an unconstrained optimization problem, which can be expressed as

$$\underset{\rho}{\text{minimize}} \quad g(\rho) = -\mathcal{J}(\rho) \tag{15}$$

where $g(\rho)$: $(0,1) \to \mathbb{R}^+$, $\mathbb{R}^+$ denotes the set of positive integers. To facilitate the optimization analysis of $\mathcal{J}$, further relax $g(\rho)$ by assuming $(1-\rho)n_{a1} \approx n_{a1}$, To facilitate the handling of the objective function, set $\bar{\xi}_1 = P_S / \left(n_{a1} + n_{c1}\right)$, $\bar{\xi}_2 = P_S / \left(n_{a2} + n_{c2}\right)$ .Therefore, $g(\rho)$ can be rewritten as

$$g(\rho) = g_i(\rho) = r_1 e^{-\frac{k_i}{1-\rho}} + r_2 e^{-\frac{k_3}{1-\rho}} \frac{2\varsigma}{\rho} \mathbb{K}_0\left(2\sqrt{\frac{\varsigma}{\rho}}\right) \tag{16}$$

if $\tau_1 \geq \tau_2$, $i = 1$, otherwise $i = 2$, where $r_1 = R_{th,1}$, $r_2 = R_{th,2}$, $\varsigma = \frac{\xi_3}{\lambda_1 \lambda_3 \eta \bar{\xi}_2}$, $k_1 = \frac{\xi_1}{\left[p_c - (p_1 + p_2)\xi_1\right]\bar{\xi}_1 \lambda_1}$, $k_2 = \frac{\xi_2}{(p_1 - p_2 \xi_2)\bar{\xi}_1 \lambda_1}$, $k_3 = \frac{\xi_3}{\left[p_2 - (p_c + p_1)\xi_3\right]\bar{\xi}_2 \lambda_2}$.

It can be seen that $g(\rho)$ has a very complex representation, and a genetic algorithm [15] is used to solve the proposed problem, as shown below.

- Set the initialization parameters as in Algorithm 1. And calculate $\text{Fitness}(\rho_i^{(t)}) = \text{g}(\rho_i^{(t)})$. Using roulette wheel selection or tournament selection to select mating pool $P_i^{(t)}$, $P_i^{(t)} = \text{Fitness}(\rho_i^{(t)}) / \sum_{j=1}^{N} \text{Fitness}(\rho_j^{(t)})$.

- Perform crossover operation with probability p_{cr} to generate offspring by combining pairs of individuals. Obtain $\rho_i^{(t+1)} = \alpha \cdot \rho_i^{(t)} + (1-\alpha)\cdot \rho_j^{(t)}$, where $\rho_i^{(t+1)}$ and $\rho_j^{(t+1)}$ are parent individuals, $\alpha \in [0,1]$ is a random crossover factor, $\rho_i^{(t+1)} = [\rho_{i1}^{(t)},\ldots,\rho_{ik}^{(t)},\rho_{jk+1}^{(t)},\ldots,\rho_{jN}^{(t)}]$.
- Perform mutation operations with probability p_{mu} to introduce mutations: $\rho_i^{(t+1)} = \rho_i^{(t)} + \delta$, where δ is a small random perturbation. Increase the iteration count until the stopping criterion is met. The convergence criterion can be defined as $\| \nabla \mathcal{J}(\rho^{(k+1)})\|_2 \le \mathcal{S}$, where $\mathcal{S}$ is the stopping threshold, and $\|\cdot\|_2$ indicates L_2 −norm.

The proposed algorithm can be described in detail as in Algorithm 1, the time complexity per generation is $O(N)$, and the overall time complexity is $O(N\bullet G)$, where the gradient of the objective function in problem (16) can be written as

$$\nabla g_i(\rho) = r_1 \bullet \frac{v_i}{(1-\rho)^2} e^{-\frac{v_i}{1-\rho}} + r_2 \bullet \left\{ e^{-\frac{v_3}{1-\rho}} \bullet \frac{v_3}{(1-\rho)^2} \frac{\zeta}{\rho} \mathbb{K}_0(\sqrt{\frac{\zeta}{\rho}}) + e^{-\frac{v_3}{(1-\rho)^2}} \times \left[-\frac{\zeta}{\rho^2} \mathbb{K}_0(\sqrt{\frac{\zeta}{\rho}}) + \frac{\zeta}{\rho} \mathbb{K}_0^{'}(\sqrt{\frac{\zeta}{\rho}}) - \frac{\sqrt{\zeta}}{2\rho^{3/2}} \right] \right\} \quad (17)$$

Algorithm 1 The algorithm of finding the optimal PS coefficient $\rho^{(*)}$

Initialization: Set the population size N, the maximum number of generations G, stopping threshold $\mathcal{S}$, and initialize the population $\{\rho_i\}_{i=1}^N$, $t=0$ .Calculate Fitness(ρ_i) .

1: repeat

2: Selection: Calculate Fitness($\rho_i^{(0)}$), form a mating pool $P_i^{(0)}$,

3:Update: $\rho_i^{(t+1)} \leftarrow \alpha \cdot \rho_i^{(t)} + (1-\alpha)\cdot \rho_j^{(t)}$

$\rho_i^{(t+1)} \leftarrow \rho_i^{(t)} + \delta$

$\rho_i^{(t+1)} = offspring$

$t \leftarrow t+1$

4:**Until:** $\| \nabla \mathcal{J}(\rho^{(k+1)})\|_2 \le \mathcal{S}$

5: **Output** $\rho^* = \arg\min_{\rho_i} g(\rho_i^{(G)}), \mathcal{J}^* = -g(\rho^*)$

V. SIMULATION AND NUMERICAL RESULTS

In this section, Monte Carlo simulation and numerical analysis are performed for the proposed RSMA-SWIPT scheme. The model of this paper is compared with RSMA and NOMA in [7] and [8], and the parameters set in this paper as follows: the cooperation time allocation factor $1-\theta = 0.5$, the fixed target data rate $R_{th,c} = R_{th,1} = R_{th,2} = 1$ bit/s/Hz, other parameters are the same as [7]. The explanation of the labels in the simulation diagram is shown in follow: "non" represents the non-cooperative mode, NOMA and RSMA all represents the cooperative mode, "initial" means the OP analysis relative to the transmission strategy of Fig. 2, and "opt" means the OP analysis relative to the GF-RSMA-SWIPT transmission strategy of Fig. 3.

Fig.4 shows a plot of the OP of U_1 and U_2 as a transmit power (dB) function. The image shows that the OP under RSMA conditions is generally lower than that under NOMA conditions, decreasing by approximately 10^{-1}. The OP of U_2 improves sequentially in the initial allocation and the optimized (opt (sim)) scenarios, and the optimized simulation values closely match the exact values. It is noteworthy that comparing the OP of the near user in Fig.4, the OP of non-RSMA is better than that of coop-RSMA. One reason is that U_1 acts as a relay, and some of its received power is used explicitly for energy forwarding. Therefore, optimizing the ρ to improve the system's traversal rate is meaningful.

The two subplots in Fig. 5 respectively show the variations of the objective function $g(\rho)$ and the far user rate $Rate,F$ with respect to the power allocation coefficient ρ and the transmit P_S. In the left subplot, the red star indicates the point $(\rho, P_s, g(\rho)) = (0.3821, 25, -1.99978)$, where $g(\rho)$ reaches its minimum value $\rho^{(*)}$. The right subplot corresponds to the point $(\rho, P_s, Rate\ F) = (0.3821, 25, 0.999954)$. As ρ increases or decreases, the function value changes significantly, indicating the sensitivity of the optimization process.

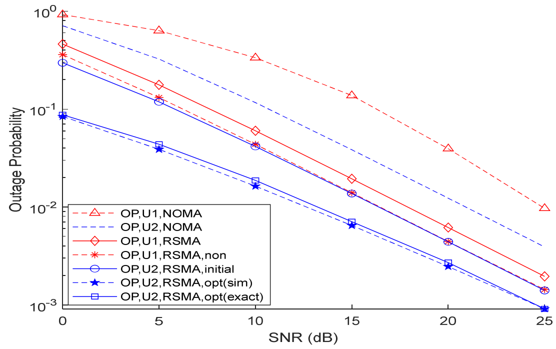

Fig. 4. OPs of U_1 and U_2, respectively, as a function of the transmit power at the source with $\rho = 0.3$.

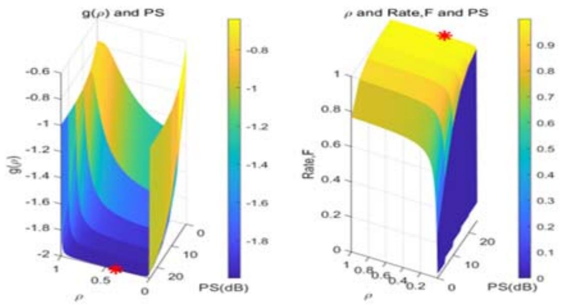

Fig. 5. Plot of P_S and ρ objective function $g(\rho)$ and traversal rate under GA algorithm.

Fig. 6 illustrates the sum-throughput with respect to the power allocation coefficient ρ at different SNR levels. It can be seen that the sum-throughput of $\rho^{(*)}$ is higher than the curves for other fixed values. The optimum point of ρ occurs between 0.3 and 0.5, which verifies the variation curves in Fig. 5. This indicates that optimizing the ρ can effectively enhance the system sum-throughput.

Fig. 7 presents the rate variations for the GF-RSMA-SWIPT, RSMA-SWIPT, and NOMA-SWIPT systems. It can be seen that the rate of U_2 corresponding to GF is higher than the original strategy. From the perspective of the sum rate, the RSMA-SWIPT system proposed in this paper shows a significant improvement compared to NOMA, with a rate increase of approximately 0.2 bit/s/Hz. As detailed in the inset, the GF consistently performs slightly better than the original strategy, confirming the superiority of the GF-RSMA-SWIPT.

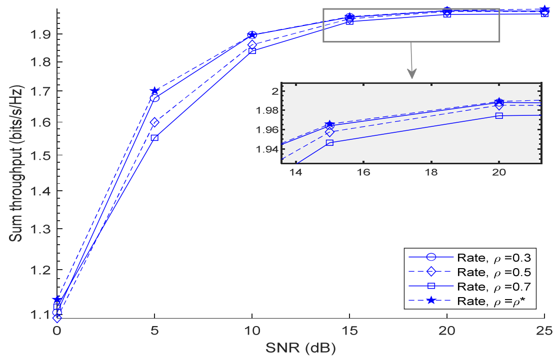

Fig. 6. Plot the traversal rate of the system for different ρ -values.

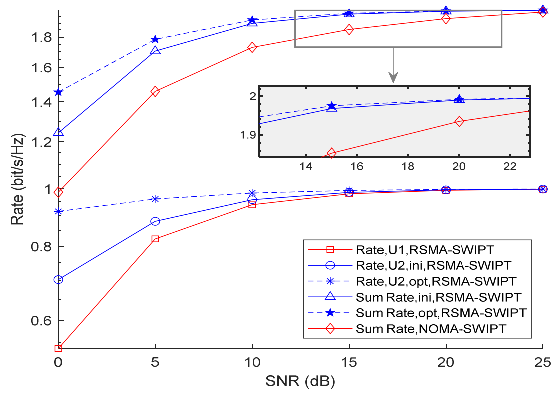

Fig. 7. Plot the comparison of system rates in different modes $(\rho = \rho^{(*)})$.

VI. Conclusion

This paper investigates the performance of the RSMA-SWIPT cooperative system in terms of OP and sum-throughput. The user's OP expressions are derived, and the optimal PS coefficients are determined using a genetic algorithm. Numerical results show that cooperative SWIPT relay transmission with the optimal coefficients improves the throughput of cell-edge users without degrading the performance of nearby users. Compared to the NOMA system, our system improves the rate by 0.2 bits/s/Hz.

Appendix A
Proof Of Theorem 2

$$\begin{aligned} P_{out}^F &= \Pr\left(\max\{\xi_{c,2},\xi\} < \xi_3\right) + \Pr\left(\max\{\xi_{c,2},\xi\} \ge \xi_3, \xi_{p,2} < \xi_4\right) \\ &= 1 - \Pr(Y \ge \sigma_1, Y \ge \sigma_2) - \Pr(cXZ \ge \xi_3)\left[\Pr(Y \ge \sigma_2) - \Pr(Y \ge \sigma_1)\right] \\ &= 1 - \int_{\max\{\sigma_1,\sigma_2\}}^{\infty} f_Y(y)dy - \Pr(XZ \ge a)\left[e^{-\frac{\sigma_2}{\lambda_2}} - e^{-\frac{\sigma_1}{\lambda_2}}\right] \end{aligned} \tag{18}$$

$$\Pr(XZ \ge a) = \iint_{XZ \ge a} \frac{1}{\lambda_1 \lambda_3} e^{-\frac{x}{\lambda_1}} \bullet e^{-\frac{z}{\lambda_3}} dxdz = \int_0^{\infty} \frac{1}{\lambda_1} e^{-\frac{x}{\lambda_1}} \bullet e^{-\frac{a}{\lambda_3 x}} dx \tag{19}$$

where $a = \xi_3 / c$, let $u = a / \lambda_3 x$ and $du = -a / \lambda_3 x^2 \ dx$, so that $dx = -a / \lambda_3 u^2 \ du$, so:

$$\Pr(XZ \ge a) = \frac{a}{\lambda_1 \lambda_3} \int_0^{\infty} e^{-\left(\frac{a}{\lambda_1 \lambda_3 u} + u\right)} \bullet u^{-2} du \tag{20}$$

let $a / \lambda_1 \lambda_3 = b$, according to the Type II modified Seibel function [14]: $\int_0^{\infty} e^{-\left(\frac{b}{u}+u\right)} \bullet u^{-2} du = 2\mathbb{K}_0\left(2\sqrt{b}\right)$, take in P_{out}^F . The original equation is proved.

ACKNOWLEDGMENT

This work was supported by NSFC, China (62071319).

References

[1] B. Clerckx, Y. Mao, R. Schober and H. V. Poor, "Rate-Splitting Unifying SDMA, OMA, NOMA, and Multicasting in MISO Broadcast Channel: A Simple Two-User Rate Analysis," in *IEEE Wireless Communications Letters*, vol. 9, no. 3, pp. 349-353, March 2020.

[2] X. Chen, F. Yan, M. Hu and Z. Lin, "Energy Efficiency Optimization of Intelligent Reflective Surface-assisted Terahertz-RSMA System," 2023 28th Asia Pacific Conference on Communications (APCC), Sydney, Australia, 2023, pp. 307-312.

[3] S. K. Singh, K. Agrawal, K. Singh and C. -P. Li, "Outage Probability and Throughput Analysis of UAV-Assisted Rate-Splitting Multiple Access," in IEEE Wireless Communications Letters, vol. 10, no. 11, pp. 2528-2532, Nov. 2021.

[4] G. Lin *et al.*, "LF-SWIPT: Outage Analysis for SWIPT Relaying Networks Using Lossy Forwarding With QoS Guaranteed," in *IEEE Internet of Things Journal*, vol. 9, no. 19, pp. 18737-18748, 1 Oct.1, 2022.

[5] Y. Mao, B. Clerckx, and Victor OK Li, "Rate-splitting multiple access for downlink communication systems: Bridging, generalizing, and outperforming SDMA and NOMA." *EURASIP journal on wireless communications and networking* 2018 (2018): 1-54.

[6] Y. Mao and B. Clerckx, "Beyond Dirty Paper Coding for Multi-Antenna Broadcast Channel With Partial CSIT: A Rate-Splitting Approach," in *IEEE Transactions on Communications*, vol. 68, no. 11, pp. 6775-6791, Nov. 2020.

[7] M. R. Camana Acosta, C. E. G. Moreta and I. Koo, "Joint Power Allocation and Power Splitting for MISO-RSMA Cognitive Radio Systems With SWIPT and Information Decoder Users," in *IEEE Systems Journal*, vol. 15, no. 4, pp. 5289-5300, Dec. 2021.

[8] J. Zhang, B. Clerckx, J. Ge and Y. Mao, "Cooperative Rate Splitting for MISO Broadcast Channel With User Relaying, and Performance Benefits Over Cooperative NOMA," in *IEEE Signal Processing Letters*, vol. 26, no. 11, pp. 1678-1682, Nov. 2019.

[9] Y. Mao, B. Clerckx and V. O. K. Li, "Rate-Splitting for Multi-User Multi-Antenna Wireless Information and Power Transfer," 2019 IEEE 20th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), Cannes, France, 2019, pp. 1-5.

[10] R. Zhang, K. Xiong, Y. Lu, P. Fan, D. W. K. Ng and K. B. Letaief, "Energy Efficiency Maximization in RIS-Assisted SWIPT Networks With RSMA: A PPO-Based Approach," in IEEE Journal on Selected Areas in Communications, vol. 41, no. 5, pp. 1413-1430, May 2023.

[11] H. Joudeh and B. Clerckx, "Sum-Rate Maximization for Linearly Precoded Downlink Multiuser MISO Systems With Partial CSIT: A Rate-Splitting Approach," in *IEEE Transactions on Communications*, vol. 64, no. 11, pp. 4847-4861, Nov. 2016,.

[12] F. Xiao, X. Li, L. Yang, H. Liu and T. A. Tsiftsis, "Outage Performance Analysis of RSMA-Aided Semi-Grant-Free Transmission Systems," in IEEE Open Journal of the Communications Society, vol. 4, pp. 253-268, 2023.

[13] Goldsmith A. Wireless communications[M]. Cambridge university press, 2005.

[14] Watson G N. A treatise on the theory of Bessel functions[M]. The University Press, 1922.

[15] J. Li and L. Li, "An Improvement Proposal of Genetic Algorithms Based on Information Entropy and Game Theory," 2019 Sixth International Conference on Social Networks Analysis, Management and Security (SNAMS), Granada, Spain, 2019, pp. 36-43.