

Statistical QoS-Aware Design of UAV-Enabled Integrated Sensing and Communication System

Zhilei Chen, Wenhao Wang and Deli Qiao

School of Communication and Electronic Engineering,

East China Normal University, Shanghai, China

Email: 51255904017@stu.ecnu.edu.cn, {whwang, dlqiao}@cee.ecnu.edu.cn

Abstract—In this paper, an unmanned aerial vehicle (UAV)-enabled integrated sensing and communication (ISAC) system over Rician fading channel operating under statistical quality of service (QoS) constraints is investigated. Effective capacity (EC) is adopted to quantify the performance of data transmissions, which represents the maximal constant arrival rate while satisfying the statistical delay requirements. Assuming the UAV remains stationary, an optimization problem of jointly designing the transmit beamforming, sensing covariance matrix, and UAV location, subjected to constraints of QoS, sensing and total transmit power, to maximize the weighted sum EC is formulated. However, the optimization problem is hard to optimally solved, because the objective function is not closed form, and UAV location variables are incorporated in exponent of steering vector. To make this issue tractable, the alternating optimization (AO) algorithm is proposed to get good quality results. Simulation results validate the superiority of the proposed scheme, particularly under stringent delay conditions.

Index Terms—Integrated sensing and communication, effective capacity, quality of service, beamforming

I. INTRODUCTION

Integrated sensing and communication (ISAC) has been regarded as a key technique to simultaneously provide communication and detection for the sixth-generation (6G) mobile communication systems [1]. Compared with traditional separated radar and communication systems, ISAC systems with reuse of the scarce spectrum and wireless infrastructure sharing can achieve higher spectral, energy, and hardware efficiency [2], which has become a prominent topic of interest.

Recently, due to adaptations of unmanned aerial vehicle (UAV) to emergency scenarios and line-of-sight (LoS) propagation links for sensing in the air-ground environment, UAV is expected to provide an aerial ISAC platform, which can be applicable for emergency scenarios, such as border reconnaissance and disaster rescue [3]–[5]. It has led to numerous novel solutions. The authors in [4] investigated a UAV-enabled integrated periodic sensing and communication system by optimizing the UAV trajectory and beamforming. Besides, the performance trade-off between sensing and computing in a UAV-enabled ISAC system was studied in [5]. Nonetheless, these system designs merely assume LoS channels for communication, which are unsuitable for the LoS-blocked scenarios, especially when the altitude of UAV is not high [6]. For tackling this disadvantage, You *et al.* incorporated Rician fading channels, which considered small-scale fading rather than only LOS channel in the UAV system [6]. However, to

the best of our knowledge, the study of the UAV-enabled ISAC over Rician fading channels is still few.

On the other hand, delay-sensitive applications like live-streaming, autonomous equipment and virtual reality of flying vehicles have emerged as key driving forces for 6G technology [7]. However, the existing literature involving UAV-enabled ISAC solely applies the ergodic Shannon capacity as metric of communication. In terms of quality of service (QoS) in communication, only constraints of signal-to-noise-ratio at the receiver and power consumption are taken into account, while delay constraints are ignored. These metrics are not sufficient for communication systems with delay-sensitive applications. To address the design of communication system under statistical delay constraints, effective capacity (EC) was proposed in [8]. Since then, EC-based design and analysis have received much attention [9], [10]. For example, Du *et al.* minimized base station (BS) usage and reduce interference in the distributed multiple-input-multiple-output system subject to statistical delay-QoS [9]. In [10], authors maximized aggregate EC in the airborne mobile wireless networks groups while guaranteeing heterogeneous statistical QoS provisioning. Unfortunately, there are few studies on delay-constrained UAV-enabled ISAC.

In this paper, we investigate the UAV-enabled ISAC system with delay constraint for data transmissions. We jointly design the transmitting beamforming, sensing covariance matrix and UAV location to maximize sum EC over Rician fading channel with statistical QoS constraints and sensing constraints. We propose an alternating optimization method to obtain solutions for first-order optimal transmitting beamforming, first-order optimal sensing covariance matrix and UAV location. Eventually, through numerical simulations, we verify the effectiveness and robustness of the proposed method about maximizing the sum EC in delay-limited ISAC system.

Notations: For matrix, the operators $(\cdot)^{-1}$, $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $\text{tr}(\cdot)$ and $\mathbb{E}(\cdot)$ stand for inverse, conjugate, transpose, Hermitian transpose, trace and expectation, respectively. $(\cdot) \succeq 0$ means that the matrix is positive semidefinite. $\|\cdot\|$ and $\|\cdot\|_F$ stands for Euclidean norm and Frobenius norm, respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we establish a model of the UAV-enabled ISAC system, introduce EC framework and formulate the optimization problem.

A. System Model

We consider a UAV-enabled ISAC system composed of one UAV as a dual-function BS equipped with M antennas, one target, and K single-antenna users. And, we assume that the position of UAV hovers at fixed height H . The BS transmits ISAC signals for detecting the target and serving the users simultaneously. In particular, the ISAC signal can be expressed as

$$\mathbf{x} = \sum_{k=1}^K \mathbf{w}_k s_k + \mathbf{s}_0, \quad (1)$$

where $\mathbf{s}_0 \in \mathbb{C}^{M \times 1}$ denotes dedicated sensing signal for the target, while $s_k \in \mathbb{C}$ and $\mathbf{w}_k \in \mathbb{C}^{M \times 1}$ denote information signal for user k , $\forall k \in \mathcal{K}$, and the corresponding beamforming vector, respectively, where \mathcal{K} denotes the set of the users.

We assume that the BS is located at $[\mathbf{q}^T, H]^T \in \mathbb{R}^{3 \times 1}$, where $\mathbf{q} = [q_x, q_y]^T$ is the horizontal position of BS, while the target and the users are located at $\mathbf{m} = [m_x, m_y]^T \in \mathbb{R}^{2 \times 1}$ and $\tilde{\mathbf{U}} = \{\mathbf{u}_1, \dots, \mathbf{u}_K\} \in \mathbb{R}^{2 \times K}$, respectively, where $\mathbf{u}_k = [u_{kx}, u_{ky}]^T \in \mathbb{R}^{2 \times 1}$, $\forall k \in \mathcal{K}$. Then, the communication channel between user k and BS can be expressed as [6]

$$\mathbf{h}_k = \sqrt{\frac{\beta}{H^2 + \|\mathbf{q} - \mathbf{u}_k\|^2}} \left(\sqrt{\frac{\alpha}{\alpha + 1}} \mathbf{a}(\mathbf{q}, \mathbf{u}_k) + \sqrt{\frac{1}{\alpha + 1}} \mathbf{h}_{\text{Rayle}} \right),$$

where $\alpha > 0$ denotes the factor of power ratio, β represents the reference channel power gain at the reference distance of 1 m, and $\mathbf{h}_{\text{Rayle}} \in \mathbb{C}^{M \times 1}$ indicates the corresponding small-scale Rayleigh fading. Besides, $\mathbf{a}(\mathbf{q}, \mathbf{u}_k) = [1, e^{j2\pi \frac{d}{\lambda} \cos \eta(\mathbf{q}, \mathbf{u}_k)}, \dots, e^{j2\pi \frac{d}{\lambda} (M-1) \cos \eta(\mathbf{q}, \mathbf{u}_k)}]^T \in \mathbb{C}^{M \times 1}$ denotes the steering vector between the BS and user k , where λ and $\eta(\mathbf{q}, \mathbf{u}_k)$ are the wavelength of the carrier and the arrival direction from user k to BS, respectively.

Considering the data transmissions, the received signal y_k at user k is

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \quad (2)$$

where $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the additional white Gaussian noise in user k . Then, the traditional Shannon capacity of user k can be given by $R_s^k = \log_2(1 + v_k)$, where

$$v_k = \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{i=1, i \neq k}^K |\mathbf{h}_k^H \mathbf{w}_i|^2 + \mathbf{h}_k^H \mathbf{R}_s \mathbf{h}_k + \sigma_k^2} \quad (3)$$

denotes the signal-to-interference-noise-ratio of user k . $\mathbf{R}_s = \mathbb{E}(\mathbf{s}_0 \mathbf{s}_0^H) \in \mathbb{C}^{M \times M}$ is the covariance matrix of the sensing signal \mathbf{s}_0 . Note that the sensing signal \mathbf{s}_0 is generated by multi-beam transmission, which means that $0 \leq \text{rank}(\mathbf{R}_s) \leq M$. The dedicated sensing signal can be recovered by eigen value decomposition (EVD) method [3].

The beam pattern gain is considered as the metric of sensing [4]. In addition to the dedicated sensing signal \mathbf{s}_0 , the communication signal \mathbf{s}_i can be exploited for sensing. Thus, the beam pattern gain $\mathcal{U}(\mathbf{R}_s, \mathbf{w}_k) \in \mathbb{R}$ is

$$\mathcal{U}(\mathbf{R}_s, \mathbf{w}_k) = \frac{1}{d^2(\mathbf{q}, \mathbf{m})} \mathbf{a}^H(\mathbf{q}, \mathbf{m}) \left(\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H + \mathbf{R}_s \right) \mathbf{a}(\mathbf{q}, \mathbf{m}),$$

where $d(\mathbf{q}, \mathbf{m})$ is the Euclidean distance between UAV and the potential target, and $\mathbf{a}(\mathbf{q}, \mathbf{m})$ is the steering vector between the BS and target.

B. Effective Capacity

To analyze the impact of delay and cater to delay-sensitive applications, we adopt EC as the metric for communication performance of this ISAC system. The EC of user k is given as

$$R_{EC}^k = - \lim_{T \rightarrow \infty} \frac{1}{\theta_k T} \log \mathbb{E}[e^{-\theta_k s(T)}], \quad (4)$$

where $s(T) = \int_0^T r_s(t) dt$ is the time-accumulated service rate in the communication duration T . And, r_s is the Shannon channel capacity. It's noted that θ is the statistical QoS exponent related to delay. According to [8], under the sufficient conditions, the queue-length random variable Q satisfies the following equation:

$$- \lim_{Q \rightarrow \infty} \frac{\log(\Pr\{Q \geq Q_{th}\})}{Q_{th}} = \theta,$$

for a certain $\theta > 0$, where Q_{th} represents a given bound for the length of queue. Then, the buffer overflow possibility related to θ can be expressed as: $\Pr\{Q \geq Q_{max}\} \approx e^{-\theta Q_{max}}$. Thus, a larger exponent θ signifies more stringent QoS requirement in communication. In this work, we assume that the data transmissions for users are subject to statistical delay constraints. In a word, EC established the maximum throughput under given QoS requirement θ .

C. Problem Formulation

Our objective is to maximize the weighted sum effective capacity of the ISAC system while ensuring the sensing task. Considering the constraints of communication, sensing and UAV location, the optimization problem is formulated as follows:

$$(P1): \max_{\{\mathbf{w}_k, \mathbf{q}, \mathbf{R}_s\} \geq 0} \sum_{k=1}^K \alpha_k R_{EC}^k \quad (5)$$

$$s.t. \mathcal{U}(\mathbf{R}_s, \mathbf{w}_k) \geq \Gamma, \quad (6)$$

$$\sum_{k=1}^K \|\mathbf{w}_k\|^2 + \text{tr}(\mathbf{R}_s) \leq P_{max}, \quad (7)$$

$$x_{min} \leq q_x \leq x_{max}, \quad (8)$$

$$y_{min} \leq q_y \leq y_{max}, \quad (9)$$

where $\alpha_k > 0$ is the weighted value of user k and Γ is the threshold set to support sensing. In (P1), the constraint (6) is to guarantee that this ISAC system can sense the target. And, the constraint (7) is the limitation of total power in the system. The constraints (8) and (9) set the feasible location region for UAV. What's more, the objective function is harnessed by the statistical QoS constraint θ .

III. PROPOSED SOLUTION FOR PROBLEM (P1)

First of all, assuming that the communication channel is block fading and the channel state information is perfect, we can simplify the objective function in problem (P1) as [11]

$$\begin{aligned} \sum_{k=1}^K \alpha_k R_{EC}^k &= \sum_{k=1}^K -\frac{\alpha_k}{\theta_k} \log \mathbb{E} \left\{ e^{-\theta_k \frac{\ln(1+v_k)}{\ln 2}} \right\}, \\ &= \log \prod_{k=1}^K \left(\mathbb{E} \left\{ (1+v_k)^{-\frac{\theta_k}{\ln 2}} \right\} \right)^{-\frac{\alpha_k}{\theta_k}}. \end{aligned}$$

As such, problem (P1) can be reformulated as

$$(P2) \quad \max_{\{\mathbf{w}_k\}, \mathbf{q}, \mathbf{R}_s \geq 0} \prod_{k=1}^K \left(\mathbb{E} \left\{ (1 + v_k)^{-\frac{\theta_k}{\ln 2}} \right\} \right)^{-\frac{\alpha_k}{\theta_k}} \quad (10)$$

s.t. (6) – (9).

Note that, due to the non-convexity of the constraint (6), the special form of the objective function and coupled variables, this problem is hard to solve optimally. Thus, we adopt alternating optimization (AO) methods to get good quality sub-optimal results of it. Specifically, the transmit beamforming and sensing covariance matrix are optimized iteratively with UAV location in an alternating manner, until the convergence of weighted sum-EC is achieved.

A. Optimization of Transmit Beamforming

We firstly optimize the transmit beamforming under a given sensing covariance matrix and UAV location. We merge all users' beamforming variables \mathbf{w}_k into a vector, i.e. $\mathbf{Vec}(\mathbf{w}) = [\mathbf{w}_1^H, \dots, \mathbf{w}_K^H]^H \in \mathbb{C}^{MK \times 1}$. Note that the constraint (7) must be satisfied with the condition of equality at the optimal result, i.e. $\mathbf{Vec}(\mathbf{w})^H \mathbf{Vec}(\mathbf{w}) + \text{tr}(\mathbf{R}_s) = P_{max}$, otherwise the remaining power can be used for communication to further increase the objective function. What's more, inspired by the transition from constrained problem into unconstrained problem [12, proposition 4], for user k , we can reformulate the part in parentheses in the expectation of (10) as

$$1 + v_k = \frac{\mathbf{Vec}(\mathbf{w})^H \mathbf{G}_k \mathbf{Vec}(\mathbf{w})}{\mathbf{Vec}(\mathbf{w})^H \mathbf{C}_k \mathbf{Vec}(\mathbf{w})},$$

where $\mathbf{G}_k \in \mathbb{C}^{MK \times MK}$ and $\mathbf{C}_k \in \mathbb{C}^{MK \times MK}$ are

$$\mathbf{G}_k = \text{blkdiag}(\mathbf{h}_k \mathbf{h}_k^H, \dots, \mathbf{h}_k \mathbf{h}_k^H) + \frac{\sigma_k^2 + \mathbf{h}_k^H \mathbf{R}_s \mathbf{h}_k}{P_{max} - \text{tr}(\mathbf{R}_s)} \mathbf{I},$$

$$\mathbf{C}_k = \text{blkdiag}(\mathbf{h}_k \mathbf{h}_k^H, \dots, \mathbf{0}, \dots, \mathbf{h}_k \mathbf{h}_k^H) + \frac{\sigma_k^2 + \mathbf{h}_k^H \mathbf{R}_s \mathbf{h}_k}{P_{max} - \text{tr}(\mathbf{R}_s)} \mathbf{I}.$$

These two matrices are both positive definite and \mathbf{I} is an identity matrix. In this transformation, if $\mathbf{Vec}(\mathbf{w})$ is changed linearly, i.e. $\alpha \mathbf{Vec}(\mathbf{w})$, the value of objective function is invariable because $1 + v_k(\alpha \mathbf{Vec}(\mathbf{w})) = 1 + v_k(\mathbf{Vec}(\mathbf{w}))$. Then, we can satisfy the constraint of power by normalization and drop (7) in (P1).

For the non-convex constraint (6), we use the technique of SCA. The LHS of (6) is in the quadratic form, which means that the first-order Taylor expansion is the tight low bound of it. Thus, we can change (6) into:

$$\zeta - \mathbf{Vec}(\mathbf{w})_{n-1}^H \mathbf{U} \mathbf{Vec}(\mathbf{w})_n \leq 0, \quad (11)$$

where $\mathbf{U} = \mathbf{D} \mathbf{D}^H$,

$$\mathbf{D} = \text{blkdiag}\{\mathbf{a}^H(\mathbf{q}, \mathbf{m}), \dots, \mathbf{a}^H(\mathbf{q}, \mathbf{m})\} \in \mathbb{C}^{MK \times K},$$

$$\zeta = d^2(\mathbf{q}, \mathbf{m}) \Gamma - \mathbf{a}^H(\mathbf{q}, \mathbf{m}) \mathbf{R}_s \mathbf{a}(\mathbf{q}, \mathbf{m}).$$

For simplicity, in the following part, $\mathbf{Vec}(\mathbf{w})$ is adopted to represent $\mathbf{Vec}(\mathbf{w})_n$.

With above transformation, (P2) can be recast into (P3):

$$(P3) : \max_{\{\mathbf{Vec}(\mathbf{w})\}} \prod_{k=1}^K \left(\mathbb{E} \left\{ \left(\frac{\mathbf{Vec}(\mathbf{w})^H \mathbf{G}_k \mathbf{Vec}(\mathbf{w})}{\mathbf{Vec}(\mathbf{w})^H \mathbf{C}_k \mathbf{Vec}(\mathbf{w})} \right)^{-\frac{\theta_k}{\ln 2}} \right\} \right)^{-\frac{\alpha_k}{\theta_k}} \quad (12)$$

s.t. $\mathbf{Vec}(\mathbf{w})_{n-1}^H \mathbf{U} \mathbf{Vec}(\mathbf{w}) \geq \zeta$.

However, the above question is still non-convex due to the non-convex objective function. In the following, we derive the first-order optimal condition of (P3).

Theorem 1: The first-order optimal transmit beamforming matrix of (P3) should satisfy

$$\mathbf{Vec}(\mathbf{w})^* = \bar{\mathbf{C}}_0^{-1} \bar{\mathbf{G}}_0 \mathbf{Vec}(\mathbf{w})^* + \lambda \bar{\mathbf{C}}_0^{-1} \mathbf{U}^T \mathbf{Vec}(\mathbf{w})_{n-1}^* \quad (13)$$

where λ is Lagrange multiplier. In the $i + 1$ th iteration, when $\lambda_{i+1} = 0$ can make (12) satisfied, λ_{i+1} should be zero. Otherwise, $\lambda_{i+1} = [\frac{\zeta - \mathbf{Z}_i(\alpha_i \bar{\mathbf{C}}_0^{-1} \bar{\mathbf{G}}_0 \mathbf{Vec}(\mathbf{w})_i^*)}{\mathbf{Z}_i(\bar{\mathbf{C}}_0^{-1} \mathbf{U}^T \mathbf{Vec}(\mathbf{w})_{n-1}^*)}]^+$, where $\alpha_i = \sqrt{\frac{P_{max} - \text{tr}(\mathbf{R}_s)}{\|\mathbf{Vec}(\mathbf{w})_i\|^2}}$ is the normalization for $\mathbf{Vec}(\mathbf{w})_i$. And, $\mathbf{Z}_{i+1} = \alpha_{i+1} \mathbf{Vec}(\mathbf{w})_{n-1}^H \mathbf{U}$. Besides, $\bar{\mathbf{G}}_0 \in \mathbb{C}^{MK \times MK}$ and $\bar{\mathbf{C}}_0 \in \mathbb{C}^{MK \times MK}$ are given in (15) and (16) respectively, where

$$\phi_i = \mathbb{E} \left\{ \left(\frac{\mathbf{Vec}(\mathbf{w})^H \mathbf{G}_i \mathbf{Vec}(\mathbf{w})}{\mathbf{Vec}(\mathbf{w})^H \mathbf{C}_i \mathbf{Vec}(\mathbf{w})} \right)^{-\frac{\theta_i}{\ln 2}} \right\}. \quad (14)$$

Proof: Please see [13] for details. \square

Theorem 2: The equation (13) always has one feasible solution by fixed point iteration algorithm.

Proof: Please see [13] for details. \square

The first-order optimal solution $\mathbf{Vec}(\mathbf{w})$ under given ϕ_i can be obtained through **Algorithm 1** as illustrated below. The optimal ϕ_i will be characterized in the following.

Algorithm 1 First-order optimal transmit beamforming

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1: Initialization:  $\mathbf{Vec}(\mathbf{w})_n$ ,  $n = 1$ 
2: repeat
3:   Initialization:  $\mathbf{V}_j = \mathbf{Vec}(\mathbf{w})_n$ ,  $j = 1$ 
4:   repeat
5:     Compute transmit beamforming by the equation (13).
6:     Normalization update  $\mathbf{V}_{j+1} = \frac{\mathbf{V}_{j+1}}{\|\mathbf{V}_{j+1}\|} \times \sqrt{P_{max} - \text{tr}(\mathbf{R}_s)}$ .
7:   until  $\frac{\|\mathbf{V}_{j+1} - \mathbf{V}_j\|}{\max(1, \|\mathbf{V}_{j+1}\|)} \leq \epsilon$  is achieved.
8:    $\mathbf{Vec}(\mathbf{w})_{n+1} = \mathbf{V}_{j+1}$ 
9: until  $\frac{\|\mathbf{Vec}(\mathbf{w})_{n+1} - \mathbf{Vec}(\mathbf{w})_n\|}{\max(1, \|\mathbf{Vec}(\mathbf{w})_{n+1}\|)} \leq \epsilon$  is achieved.
10: Output:  $\mathbf{Vec}(\mathbf{w})_{n+1}$ .
    
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B. Optimization of Sensing Covariance Matrix

Under the first-order optimal transmit beamforming $\mathbf{Vec}(\mathbf{w})$ of (P3) and UAV location, the optimization problem of sensing covariance matrix is turned into:

$$(P4) : \max_{\{\mathbf{R}_s \geq 0\}} \prod_{k=1}^K \left(\mathbb{E} \left\{ (1 + v_k)^{-\frac{\theta_k}{\ln 2}} \right\} \right)^{-\frac{\alpha_k}{\theta_k}} \quad (15)$$

s.t. (6), (7).

The above problem is again non-convex and challenging to solve. Fortunately, we find an important property that can be used to drop the constraint of sensing and use normalization to

$$\bar{\mathbf{G}}_0 = \sum_{i=1}^K \frac{\alpha_i}{\ln 2} \phi_i^{-1} \prod_{k=1}^K \phi_k^{-\frac{\alpha_k}{\theta_k}} \left(\frac{\text{Vec}(\mathbf{w}^H) \mathbf{G}_i \text{Vec}(\mathbf{w})}{\text{Vec}(\mathbf{w})^H \mathbf{C}_i \text{Vec}(\mathbf{w})} \right)^{-\frac{\theta_i}{\ln 2}} \frac{\mathbf{G}_i^T}{\text{Vec}(\mathbf{w})^H \mathbf{G}_i \text{Vec}(\mathbf{w})} \quad (15)$$

$$\bar{\mathbf{C}}_0 = \sum_{i=1}^K \frac{\alpha_i}{\ln 2} \phi_i^{-1} \prod_{k=1}^K \phi_k^{-\frac{\alpha_k}{\theta_k}} \left(\frac{\text{Vec}(\mathbf{w}^H) \mathbf{G}_i \text{Vec}(\mathbf{w})}{\text{Vec}(\mathbf{w})^H \mathbf{C}_i \text{Vec}(\mathbf{w})} \right)^{-\frac{\theta_i}{\ln 2}} \frac{\mathbf{C}_i^T}{\text{Vec}(\mathbf{w})^H \mathbf{C}_i \text{Vec}(\mathbf{w})} \quad (16)$$

satisfy it. To be more specific, we state the fully beam pattern gain property in the following Theorem:

Theorem 3: The inequality constraint of sensing (6) must be satisfied with equality at the first-order optimal result.

Proof: Please see [13] for details. \square

Moreover, because of the positive semidefinite property of sensing covariance matrix, we consider this transformation $\mathbf{R}_s = \mathbf{T}^H \mathbf{T}$, where $\mathbf{T} \in \mathbb{C}^{M \times M}$ is a auxiliary variable. Through the Lemma 3 and borrowing the principle of transformation form (P1) to (P3), the (P4) can be recast into (P5):

$$(P5) : \max_{\{\mathbf{T}\}} \prod_{k=1}^K \left(\mathbb{E} \left\{ \left(\frac{\text{tr}(\mathbf{T} \mathbf{Q}_k \mathbf{T}^H)}{\text{tr}(\mathbf{T} \mathbf{P}_k \mathbf{T}^H)} \right)^{-\frac{\theta_k}{\ln 2}} \right\} \right)^{-\frac{\alpha_k}{\theta_k}} \\ s.t. \quad \text{tr}(\mathbf{T}^H \mathbf{T}) \leq P_{max} - \text{Vec}(\mathbf{w})^H \text{Vec}(\mathbf{w}),$$

where $\mathbf{P}_k = \psi_k \mathbf{a}(\mathbf{q}, \mathbf{m}) \mathbf{a}^H(\mathbf{q}, \mathbf{m}) + \mathbf{h}_k \mathbf{h}_k^H$ and $\mathbf{Q}_k = \varphi_k \mathbf{a}(\mathbf{q}, \mathbf{m}) \mathbf{a}^H(\mathbf{q}, \mathbf{m}) + \mathbf{h}_k \mathbf{h}_k^H$ are both semipositive definite matrices, where $\psi_k = \frac{\sum_{i=1, i \neq k}^K |\mathbf{h}_k^H(\mathbf{q}, \mathbf{u}_k) \mathbf{w}_i|^2 + \sigma_k^2}{d^2(\mathbf{q}, \mathbf{m}) \Gamma - \mathbf{a}^H(\mathbf{q}, \mathbf{m}) (\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H) \mathbf{a}(\mathbf{q}, \mathbf{m})}$ and $\varphi_k = \frac{\sum_{i=1}^K |\mathbf{h}_k^H(\mathbf{q}, \mathbf{u}_k) \mathbf{w}_i|^2 + \sigma_k^2}{d^2(\mathbf{q}, \mathbf{m}) \Gamma - \mathbf{a}^H(\mathbf{q}, \mathbf{m}) (\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H) \mathbf{a}(\mathbf{q}, \mathbf{m})}$. Then, we can derive the first-order optimal conditions of \mathbf{T} :

Theorem 4: The first-order optimal auxiliary variable \mathbf{T} of (P5) should satisfy

$$\mathbf{T} = \mathbf{T} \bar{\mathbf{Q}}_n (\bar{\mathbf{P}}_n + \lambda_1 \mathbf{I})^{-1}, \quad (17)$$

where $\bar{\mathbf{Q}}_n \in \mathbb{C}^{M \times M}$ and $\bar{\mathbf{P}}_n \in \mathbb{C}^{M \times M}$ are given as follows,

$$\bar{\mathbf{Q}}_n = \sum_{k=1}^K \frac{\alpha_k}{\ln 2} \phi_k^{-1} \prod_{i=1}^K \phi_i^{-\frac{\alpha_i}{\theta_i}} \left\{ \frac{\text{tr}(\mathbf{T} \mathbf{Q}_k \mathbf{T}^H)}{\text{tr}(\mathbf{T} \mathbf{P}_k \mathbf{T}^H)} \right\}^{-\frac{\theta_k}{\ln 2}} \frac{\mathbf{Q}_k}{\text{tr}(\mathbf{T} \mathbf{Q}_k \mathbf{T}^H)}, \\ \bar{\mathbf{P}}_n = \sum_{k=1}^K \frac{\alpha_k}{\ln 2} \phi_k^{-1} \prod_{i=1}^K \phi_i^{-\frac{\alpha_i}{\theta_i}} \left\{ \frac{\text{tr}(\mathbf{T} \mathbf{Q}_k \mathbf{T}^H)}{\text{tr}(\mathbf{T} \mathbf{P}_k \mathbf{T}^H)} \right\}^{-\frac{\theta_k}{\ln 2}} \frac{\mathbf{P}_k}{\text{tr}(\mathbf{T} \mathbf{P}_k \mathbf{T}^H)}.$$

where λ_1 is Lagrange multiplier. When $\bar{\mathbf{P}}_n$ is invertible and $\lambda_1 = 0$ can make (7) satisfied, $\lambda_1 = 0$. Otherwise, in the $i + 1$ th iteration, $\lambda_1^{i+1} = \beta_i \times \sqrt{\frac{\text{tr}(\mathbf{W}_i^H \mathbf{W}_i)}{P_{max} - \|\text{Vec}(\mathbf{w})\|^2}}$, where $\beta_i = \sqrt{\frac{d^2 \Gamma - \mathbf{a}^H (\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H) \mathbf{a}}{\mathbf{a}^T \mathbf{T}_i^H \mathbf{T}_i \mathbf{a}}}$ is the normalization factor for \mathbf{T}_i , and $\mathbf{W}_i = \beta_i \mathbf{T}_i (\bar{\mathbf{Q}}_n - \bar{\mathbf{P}}_n)$. And, there is always one feasible solution in the equation (17) by the fixed point iteration.

Proof: Please see [13] for details. \square

Thus, we can get the first-order optimal sensing covariance matrix as following **Algorithm 2**.

C. The optimization of UAV's placement

In this part, we exploit the mobility of UAV to enhance the performance of UAV-enabled ISAC system. Under given

Algorithm 2 First-order optimal sensing covariance matrix

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1: Initialization:  $\mathbf{T}_j, j = 0$ 
2: repeat
3:   Set  $j = j + 1$ .
4:   Compute sensing covariance matrix by the equation (17).
5:   Normalization update  $\mathbf{T}_j \times \sqrt{\frac{d^2 \Gamma - \mathbf{a}^H (\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H) \mathbf{a}}{\mathbf{a}^T \mathbf{T}_j^H \mathbf{T}_j \mathbf{a}}}$ .
6: until  $\frac{\|\mathbf{T}_j - \mathbf{T}_{j+1}\|_F}{\max(1, \|\mathbf{T}_{j+1}\|_F)} \leq \epsilon$  is achieved.
7: Output:  $\mathbf{R} = \mathbf{T}_{j+1}^H \mathbf{T}_{j+1}$ .
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$\{\text{Vec}(\mathbf{w}), \mathbf{R}_s\}$, the optimization problem of UAV location is given by

$$(P6) : \max_{\{\mathbf{q}\}} \sum_{k=1}^K -\frac{\alpha_k}{\theta_k} \log \mathbb{E} \{ e^{-\theta_k \log(1 + \gamma_k)} \} \\ s.t. \quad (6), (8), (9).$$

We approximate the objective function to be more tractable instead of using time-consuming ergodic method.

Proposition 1: The approximation of EC in the objective function of (P8) is given as follows:

$$\log \mathbb{E} \{ e^{-\theta_k \log(1 + \gamma_k)} \} \\ \leq \log \left(1 + \frac{\mathbb{E} [\|\mathbf{w}_k\|^2]}{\sum_{i \neq k}^K \mathbb{E} [\|\mathbf{w}_i\|^2] + \mathbb{E} [\text{tr}(\mathbf{R}_s)] + \frac{d^2}{\beta} \frac{\alpha+1}{\alpha M+1} \sigma^2} \right). \quad (18)$$

Proof: Please see [13] for details. \square

As for the non-convex constraint (6), we apply the SCA technique and introduce the slack variants,

$$v \geq H^2 + \|\mathbf{q} - \mathbf{m}\|^2, u_k \geq H^2 + \|\mathbf{q} - \mathbf{u}_k\|^2, \forall k. \quad (19)$$

Then, we get the following transformation [3]:

$$h^{(l)} - z^{(l)}(v - v^{(l)}) \geq \Gamma \times v \quad (20)$$

where l stands for the l -th iteration. $h^{(l)}$ and $z^{(l)}$ are given as follows:

$$h^{(l)} = \sum_{m=1}^M \mathbf{W}_{m,m} + 2 \sum_{p=1}^M \sum_{q=p+1}^M |\mathbf{W}_{p,q}| \cos(\theta_{p,q}^{\mathbf{W}} + 2\pi \frac{d}{\lambda} (q-p) \frac{H}{\sqrt{v}}), \\ z^{(l)} = -4\pi \sum_{p=1}^M \sum_{q=p+1}^M |\mathbf{W}_{p,q}| \sin(\theta_{p,q}^{\mathbf{W}} + 2\pi \frac{d}{\lambda} (q-p) \\ \times \frac{H}{\sqrt{v}}) \frac{d(q-p)}{\lambda} \frac{H}{v^{\frac{3}{2}}},$$

where $\mathbf{W} = \sum_{i=1}^K \mathbf{w}_i \mathbf{w}_i^H + \mathbf{R}_s$ and $|\mathbf{W}_{p,q}|$ represents the norm of the element in the p -th row and the q -th column of \mathbf{W} . $\theta^{\mathbf{W}}$ is the phase of one element in \mathbf{W} . It's remarkable that \mathbf{W} we used there is the expectation of \mathbf{W} obtained by

$$\mathbb{S} = \frac{1}{\ln 2} \frac{-\mathbb{E}[\|w_k\|^2] \frac{\sigma^2}{\beta} \frac{\alpha+1}{\alpha M+1}}{\left(\sum_{i \neq k}^K \mathbb{E}[\|w_i\|^2] + \mathbb{E}[tr(R_s)] + \frac{u_k}{\beta} \frac{\alpha+1}{\alpha M+1} \sigma^2\right) \left(\sum_{i=1}^K \mathbb{E}[\|w_i\|^2] + \mathbb{E}[tr(R_s)] + \frac{u_k}{\beta} \frac{\alpha+1}{\alpha M+1} \sigma^2\right)} \quad (21)$$

the above optimization of transmit beamforming and sensing covariance matrix.

However, the objective function is still non-convex. Thus, applying the SCA techniques on the objective function, we can get

$$(P7) : \max_{\{q, v, u_k\}} \sum_{k=1}^K \frac{\alpha_k}{\ln 2} \left(\mathbb{F}(u_k^{(l)}) + \mathbb{S}(u_k^{(l)}) (u_k - u_k^{(l)}) \right) \\ \text{s.t. (8), (9), (19), (20).}$$

where $\mathbb{F}(u_k) = \log \left(1 + \frac{\mathbb{E}[\|w_k\|^2]}{\sum_{i \neq k}^K \mathbb{E}[\|w_i\|^2] + \mathbb{E}[tr(R_s)] + \frac{u_k}{\beta} \frac{\alpha+1}{\alpha M+1} \sigma^2} \right)$, $\mathbb{S}(u_k)$ shows in top of the page, and l stands for the l -th iteration during SCA. We can find that the objective function of (P7) is a convex function of the slack variant u_k . And, all constraints is convex. Thus, the problem (P7) can be solved by CVX tools.

D. The overall algorithm for (P1)

Finally, we adopt the AO algorithm to optimize the transmit beamforming, sensing covariance matrix and UAV location to solve the problem (P1). For the unknown variable ϕ in in the **Algorithm 1** and **Algorithm 2**, the domain of definition and range in ϕ are both within $(0, 1)$, which makes the fixed point theory can be applied to obtain ϕ [14]. Under a given ϕ and q , we optimize $\text{Vec}(w)$ and R_s alternatively until the power of radar is almost unchanged. Thus, **Algorithm 3** gives the whole algorithm for the design of transmit beamforming and sensing covariance matrix under given UAV location.

Algorithm 3 Proposed Algorithm for transmit beamforming and sensing covariance matrix in ISAC under given UAV location

```

1: Initialization:  $k = 0$ ,  $L$ , iteration variables  $\phi^{(k)} = [1, \dots, 1]$ .
2: repeat
3:   Initialization:  $\text{Vec}(w)^0$  and  $R_s^0$  that meet the constraints
4:   repeat
5:     Set  $j = 0$ .
6:     With given  $R_s^j$ , get  $\text{Vec}(w)^{j+1}$  by Algorithm1.
7:     With given  $\text{Vec}(w)^{j+1}$ , get  $R_s^{j+1}$  by Algorithm2.
8:   until  $\frac{tr(R_s^{j+1} - R_s^j)}{tr(R_s^{j+1})} \leq \epsilon$  or the maximal iteration number is achieved.
9:   Set  $k = k + 1$ .
10:  Obtain the transmit beamforming and sensing covariance matrix for  $L$  communication channel.
11:  Calculate  $\phi_k$  from (14).
12: until  $\|\phi_k - \phi_{k-1}\| \leq \epsilon$  is achieved.
```

In UAV deployment optimization problem, for assuring the convergence and accuracy of this method, we introduce the trusted region constraint into above problem, i.e. $\|q^{(k)} - q^{(k-1)}\|^2 \leq \chi$. If the value of objective function in k -th iteration is less than that of $k-1$ -th iteration, χ will be dwindled. The whole algorithm of statistical QoS-aware design of UAV-Enabled ISAC system is given in the **Algorithm 4**.

Algorithm 4 Statistical QoS-Aware Design of UAV-enable ISAC System

```

1: Initialization: the location of UAV  $q^0$ .
2: repeat
3:   Initialization:  $k = 0$ ,  $L$ , iteration variables  $\phi^{(k)} = [1, \dots, 1]$ .
4:   With given UAV location  $q_k$ , get first-order optimal weighted sum EC  $ec$  by the proposed Algorithm 3.
5:   repeat
6:     Initialization:  $l = 0$ 
7:     obtain  $q^{l+1}$  by solving the problem (P10) with cvx.
8:      $l = l + 1$ 
9:   until  $\|q^l - q^{l-1}\|^2 \leq \epsilon$  or the maximal iteration number is achieved.
10:   $k = k + 1$ 
11:  if the value of objective function  $ec$  in (P8) is less than the previous iteration,  $\chi = \chi/5$ . Until  $ec$  increases,  $q^{k+1} = q^k$ 
12: until  $|ec^k - ec^{k-1}| \leq \epsilon$  is achieved.
```

For showing the convergence of **Algorithm 4**, first and foremost, we have the following proposition to show the convergence of **Algorithm 3**:

Proposition 2: The value of objective function of (P1) is non-decreasing over the iterations by applying the **Algorithm 3** in inner layer of **Algorithm 4**.

Proof: Please see [13] for details. \square

In the outer layer, this convergence can be assured by the fixed point iteration [14]. Thus, this proposed **Algorithm 3** is guaranteed to converge. As for the location optimization, when χ become small, the UAV location optimization in the inner iteration of **Algorithm 4** can be guaranteed as be convergent [3]. And, due to the bounded region and power, the outer iteration can converge. Consequently, the convergence of **Algorithm 4** can be assured. The total complexity of **Algorithm 4** is $O(L_{ot1} L_{ot} L_{in} (L_1 (MK)^3 + L_2 M^3))$, where L_{ot1} , L_{ot} , L_{in} , L_1 and L_2 stand for the number of iteration in the outer layer in **Algorithm 4**, the outer and inner layer in the **Algorithm 3**, **Algorithm 1** and **Algorithm 2**, respectively, because the complexity of **Algorithm 4** is determined by computing the transmit beamforming in step 6 and computing the sensing covariance matrix in step 7 of **Algorithm 3**.

IV. SIMULATION

In this section, the numerical results are provided to show the performance of the proposed algorithm. There are 4 users with one single antenna and 1 target. The feasible region of UAV is $0 \text{ m} \leq q_x \leq 100 \text{ m}$, $0 \text{ m} \leq q_y \leq 200 \text{ m}$. The channel factor α is 2 unless otherwise specified. Moreover, according to the large number lemma, $L = 10^3$ communication channels are used to calculate ϕ_i . The remaining system parameters are set as: $H = 30 \text{ m}$, $M = 8$, $\beta_0 = -20 \text{ dB}$, $\sigma^2 = -80 \text{ dBm}$, $\alpha_i = 1, \forall i$, $u_1 = (280, 200) \text{ m}$, $u_2 = (300, 200) \text{ m}$, $u_3 = (320, 200) \text{ m}$, $u_4 = (340, 200) \text{ m}$, $m = (10, 100) \text{ m}$, and $P_{max} = 10 \text{ dBw}$. For comparison, we consider six bench-

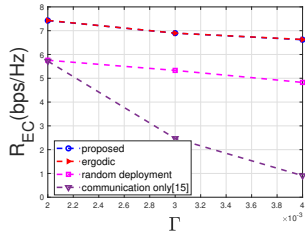
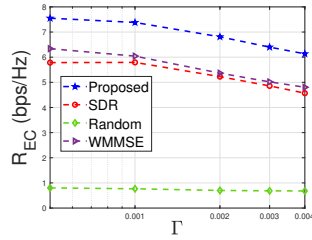
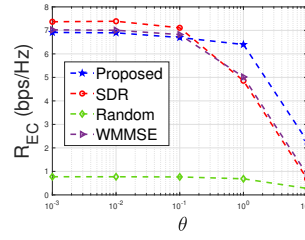
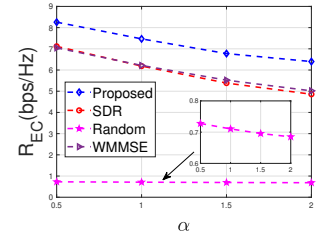


Fig. 1: Proposed method vs ergodic method


 Fig. 2: Sum EC R_{EC} vs Threshold Γ

 Fig. 3: Sum EC R_{EC} vs QoS exponent θ

 Fig. 4: Sum EC R_{EC} vs Channel Factor α

mark schemes, namely, 1) ergodic method, which splits up the feasible region into small grids and finds the best location of maximizing the weighted EC; 2) random deployment scheme, where UAV is deployed randomly; 3) communication only scheme, which only considers the communication performance and LOS channel and finds the location as [15, III.A]; 4) semidefinite relaxation (SDR) scheme, which only considers the Shannon capacity and use the techniques of SDR and SCA to maximize the Shannon capacity as [3]; 5) random scheme, which use the initialization of sensing covariance matrix in (P1) and transmit beamforming obtained by WMMSE; 6) WMMSE scheme, which replace **Algorithm 1** in **Algorithm 3** with WMMSE [12].

In Fig. 1, we assume $\theta_i = 0.01, \forall i$. We can find that the weighted sum EC obtained by proposed method is not far apart from the ergodic method and better than other benchmarks. However, the ergodic method is time-consuming. Thus, the left will not present the result of ergodic method. In Fig. 2, we assume $\theta_i = 1, \forall i$. We can find that the proposed method achieves better performance than other methods. Also, we can see that the sum EC decreases as the threshold of sensing increases. When the threshold of sensing is less than 1×10^{-3} , the sum EC is invariant. That's because that the threshold is so low that the ISAC system can simply use the communication signal to fulfill the sensing task. In Fig. 3, we assume $\Gamma = 3 \times 10^{-3}$. We can find that, when the QoS exponent is less than 0.01, the sum EC stays almost invariant since the delay constraint is loose. In this condition, the problem of (P1) degenerates into maximize the expectation of Shannon capacity. The SDR method outperforms our proposed method, because it is to obtain the upper bound of capacity of different channels by cvx tools. It is interesting that the proposed method can achieve larger sum EC than WMMSE-based method and SDR method when the delay constraint is stringent. Meanwhile, in Fig.4, as the channel factor increases, we can see that the sum EC drops. That's because the locations of user we set are close to each other, which introduces strong interference among users, whereas small-scale fading will be helpful for data transmissions.

V. CONCLUSION

In this paper, we have investigated a UAV-enabled ISAC system under the statistical QoS constraints. We have employed the EC as throughput metric for this ISAC system and assumed

the Rician fading channel for communication. Then, we have formulated the optimization problem aiming at maximizing the EC subject to the total power, sensing, and UAV location constraints. We have proposed the algorithm based AO method and achieved good quality results of the original problem through solving the sub-problems alternatively. Numerical results have verified that the proposed method can provide an improved performance gain and trade-off to the ISAC system.

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