On the Fault-Tolerant Capability of Optimal Constructions of Optical Priority Queues

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***Abstract*—Construction of optical buffers for packet conflict resolution is one of the most important and challenging issues in all-optical packet switching. Priority queues (PQs) is one of the most general and versatile buffering schemes, and includes the most commonly used first-in first-out (FIFO) queues and last-in first-out (LIFO) queues as special cases. The best constructions of optical PQs currently available was obtained by Cheng *et al.* by using a feedback architecture consisting of an optical crossbar switch and k groups of optical FIFO multiplexers with delay one (FM1's), where the i^{th} group has m_i parallel optical n_i-to-1 FM1's (n_iFM1's) with the same buffer size B_i ($B_i \geq 1$) for $i = 1, 2, \ldots, k$ (see Figure 2 in Section I). In this paper, we consider an important practical issue in the constructions of optical PQs: the *fault-tolerant* capability. We consider the scenario that each group of FM1's has the same number of FM1's, say, $m_i = m$ for $1 \leq i \leq k$, and show that the *optimal* constructions obtained by Cheng *et al.* possess fault-tolerant capability: the feedback architecture can still be operated as an optical PQ but with a *smaller* buffer size after up to f FM1's fail to function properly, where the fault-tolerant capability f can be expressed in terms of the number m of FM1's in each group and the numbers $n_1, n_2, \ldots, n_k$ of arrival links of the FM1's in the k groups as $f = \min_{1 \leq i \leq k} \lfloor (m-1)/(n_i - 1) \rfloor$. Such a result can be used in the design of the parameters m and $n_1, n_2, \ldots, n_k$ to provide quality of service (QoS) that guarantees a certain level of fault-tolerant capability.**

***Index Terms*—FIFO multiplexers, optical buffers, optical queues, optical switches, priority queues.**

I. Introduction

Constructions of optical buffers for packets conflict resolution is one of the most important and challenging issues in all-optical packet switching. A popular and feasible approach for the implementation of optical buffers is to use optical fiber delay lines (FDLs) as the storage media to store optical packets and use optical crossbar switches to route optical packets through the optical FDLs [1]. By carefully designing the routing policy performed by the optical crossbar switches and carefully choosing the delays of the optical FDLs, one can route optical packets to the right places at the right times and achieve *exact* emulations of the desired optical buffers.

In the last thirty years, there have been extensive works on Switched-Delay-Lines (SDL) constructions of optical buffers. These works include output-buffered switches, first-in first-out (FIFO) multiplexers, FIFO queues, last-in first-out (LIFO) queues, PQs, time slot interchanges, linear compressors/decompressors, non-overtaking delay lines, and flexible delay lines, and FIFO/LIFO/absolute contractors. Due to space constraint, we only list the references [2]–[13] on optical PQs. Moreover, results on the fundamental complexity of SDL constructions of optical queues can be found in [14] and performance analysis for optical queues has been addressed in [15] and [16]. For review articles on SDL constructions of optical buffers as well as related implementation and feasibility issues, we refer to [17]–[22] and the references therein.

In this paper, we consider SDL constructions of optical PQs. Priority queues is one of the most general and versatile buffering schemes, and includes the most commonly used FIFO queues and LIFO queues as special cases, where the arrival time of a packet is used for the assignment of its priority. In a PQ, each packet is associated with a *priority* upon its arrival, the packet with the *highest* priority is sent out from the queue whenever there is a departure request and there are packets in the queue, and the packet with the *lowest* priority is dropped from the queue whenever there is a buffer overflow.

Sarwate and Anantharam of UC Berkeley [2] are the first to propose constructions of optical PQs by using a feedback architecture consisting of an optical $(M+2)\times(M+2)$ crossbar switch and M optical FDLs with properly chosen delays $d_1, d_2, \ldots, d_M$ (see Figure 1). The buffer size achieved in [2] is $O(M^2)$. By using better designs of the routing policy performed by the optical crossbar switch and better choices of the delays of the optical FDLs in Figure 1, several improvements on the $O(M^2)$ buffer size obtained in [2] have been made in [3]–[13]. The best result currently available was obtained by Cheng *et al.* in [11, Theorem 10] and [13, Theorem 12], and the buffer size achieved in [11] and [13] is $2^{O(\sqrt{\alpha M})}$, where α is a constant that depends on the parameters used in their constructions. The buffer size $2^{O(\sqrt{\alpha M})}$ is "exponential" in $\sqrt{M}$ and significantly outperforms all previous results in [2]–[10] that are only "polynomial" in M.

The constructions in [13] use a feedback architecture consisting of an optical crossbar switch and k groups of optical FIFO multiplexers with delay one (FM1's), where the i^{th} group

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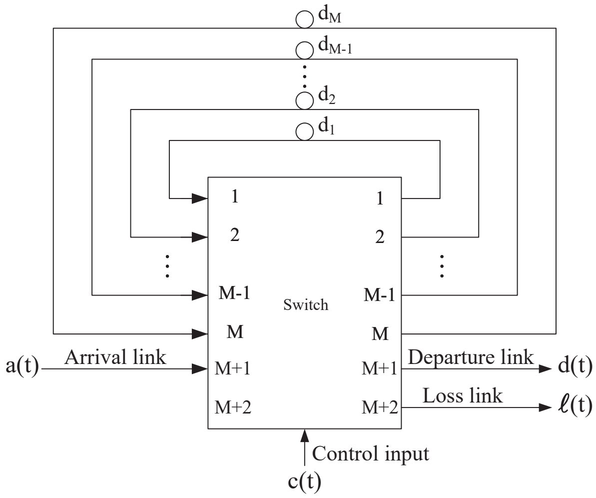

Fig. 1. A construction of an optical PQ by using a feedback architecture consisting of an optical $(M+2) \times (M+2)$ crossbar switch and M FDLs with delays $d_1, d_2, \ldots, d_M$.

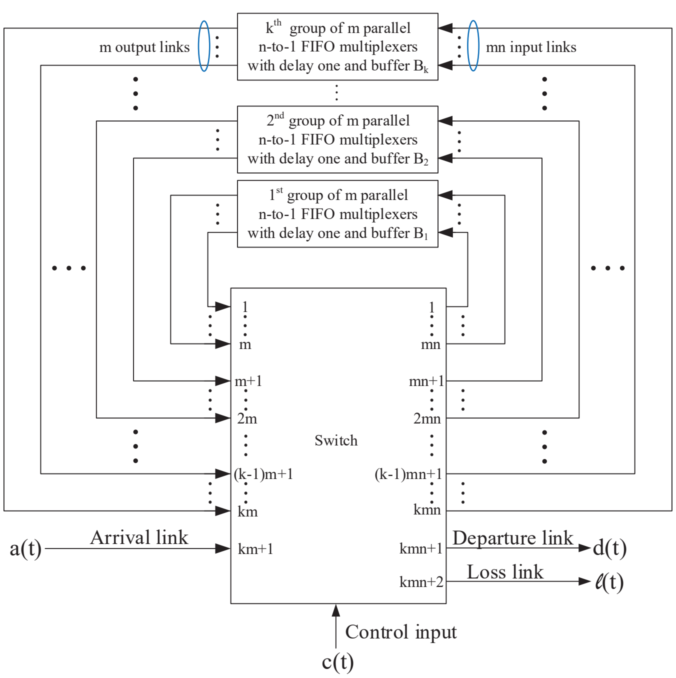

Fig. 2. A construction of an optical PQ by using a feedback architecture consisting of an optical $(\Sigma_{i=1}^{k} m_i n_i + 2) \times (\Sigma_{i=1}^{k} m_i n_i + 2)$ crossbar switch and k groups of optical FIFO multiplexers with delay one (FM1's), where the i^{th} group has m_i parallel optical n_i-to-1 FM1's (n_iFM1's) with the same buffer size B_i ($B_i \geq 1$) for $i = 1, 2, \ldots, k$. For brevity, in this figure we denote $M'_i = \Sigma_{j=1}^{i} m_j$ and $M''_i = \Sigma_{j=1}^{i} m_j n_j$ for $i = 1, 2, \ldots, k$.

has m_i parallel optical n_i-to-1 FM1's (n_iFM1's) with the same buffer size B_i ($B_i \geq 1$) for $i = 1, 2, \ldots, k$ (see Figure 2).

In this paper, we address an important practical issue in the constructions of optical PQs: the *fault-tolerant* capability. We consider the scenario that each group of FM1's has the same number of FM1's, say, $m_i = m$ for $1 \leq i \leq k$, and use the general constructions of optical PQs obtained in [13] to show that the optimal construction in [13] possesses fault-tolerant capability so that the feedback architecture can still be operated as an optical PQ but with a *smaller* buffer size after up to f FM1's fail to function properly, where the fault-tolerant capability f can be expressed in terms of the number m of FM1's in each group and the numbers $n_1, n_2, \ldots, n_k$ of arrival links of the FM1's in the k groups as $f = \min_{1 \leq i \leq k} \lfloor (m-1)/(n_i - 1) \rfloor$ (see Theorem 3 in Section III). Such a result can be used in the design of the parameters m and $n_1, n_2, \ldots, n_k$ to provide quality of service (QoS) that guarantees a certain level of fault-tolerant capability.

The rest of the paper is organized as follows. In Section II, we give a formal definition of a PQ and briefly review the constructions of optical PQs in [13]. In Section III, we show that the optimal constructions in [13] possess fault-tolerant capability, and express the fault-tolerant capability in terms of the parameters used in the constructions. Finally, we conclude this paper in Section IV.

II. Review of the Constructions of Optical Priority Queues by Cheng *et al.*

In this section, we first give a formal definition of a PQ, and then review the constructions of optical PQs in [13].

A. Priority Queues

As in most works on SDL constructions of optical queues, in this paper we consider a discrete-time setting and make the following assumptions: (i) Time is *slotted* and *synchronized*. (ii) Packets are of the *same* size so that a packet can be transmitted through a link within a time slot. (iii) An optical $M \times M$ crossbar switch is a network element with M input links and M output links that can realize all of the $M!$ *permutations* between its inputs and its outputs. (iv) An FDL with delay d is a network element with one input link and one output link that requires d time slots for a packet to traverse through. (v) There is at most one packet from any link of any network element at any time slot. (vi) Every network element is initially empty at time slot $t = 0$.

We note that *variable*-size packets can be taken care of with ease by implementing packet segmentation at the sources and packet reassembly at the destinations.

To be concise, in the rest of this paper we simply refer to time slot t as "slot t." Since there can be at most one packet from any link at any slot t (by assumption (v)), we characterize a link by its link state and say that a link is in state 1 (resp., state 0) at slot t if there is a packet (resp., there is no packet) from that link at slot t.

A PQ with buffer size B is a network element with one arrival link, one departure link, one loss link, and one control

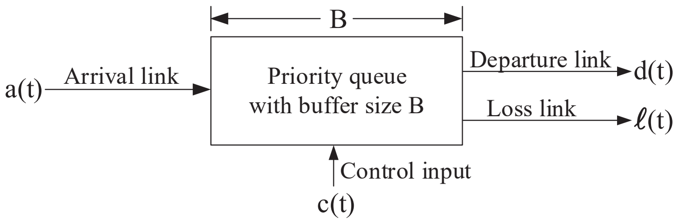

Fig. 3. A priority queue with buffer size B.

input (see Figure 3). We denote $a(t)$, $d(t)$, and $\ell(t)$ as the link states of the arrival link, the departure link, and the loss link, respectively, at slot t. We denote $c(t) = 1$ (resp., $c(t) = 0$) if there is a departure request (resp., there is no departure request) from the controller at slot t. We also denote $q(t)$ as the number of packets stored in the buffer of the PQ at slot t.

At each slot t, each of the packets in the PQ at slot t, including the packets buffered in the queue at slot $t-1$ and the packets from the arrival link at slot t, is assigned a *distinct* priority. The priority is assigned in such a way that the priority of the arrival packet (if any) at slot t can be *arbitrarily* assigned and the *relative* priority order between any two packets (if any) buffered in the queue at slot $t-1$ remains unchanged at slot t.

We define a PQ with buffer size B by the following three properties:

(P1) Nonidling and *priority departure:* If there is a departure request from the controller and there are packets in the queue at slot t, i.e., $c(t) = 1$ and $q(t-1) + a(t) \geq 1$, then there is a departure packet at slot t, and the departure packet at slot t is the packet in the queue at slot t with the *highest* priority. Otherwise, there is no departure packet at slot t.

(P2) Maximum buffer usage and *priority loss:* If there is a buffer overflow at slot t, i.e., $c(t) = 0$, $q(t-1) = B$, and $a(t) = 1$, then there is a loss packet at slot t,, and the loss packet at slot t is the packet in the queue at slot t with the *lowest* priority. Otherwise, there is no loss packet at slot t.

(P3) Flow conservation: Packets in the queue at slot t are either buffered in the queue at slot t or transmitted through the departure link or the loss link at slot t.

B. The Constructions of Optical Priority Queues by Cheng et al.

We recall the following two results from [13]. The first result tells us, for given $1 \leq s \leq k-1$ and $m_1, m_2, \ldots, m_k \geq 1$, how to choose the parameters $n_1, n_2, \ldots, n_k$, $B_1, B_2, \ldots, B_k$, and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ so that the feedback architecture in Figure 2 can be operated as an optical PQ under the priority-based routing policy in [13] (we note that the parameters s and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ are used in the priority-based routing policy in [13].).

Theorem 1 *[13, Theorem 1] Suppose that $1 \leq s \leq k-1$ and $m_1, m_2, \ldots, m_k \geq 1$. Then the feedback architecture in Figure 2 can be operated as an optical PQ with buffer size $U_k = \Sigma_{i=1}^{k} |\Psi_i|$ under the priority-based routing policy in [13] if the parameters $n_1, n_2, \ldots, n_k$, $B_1, B_2, \ldots, B_k$, and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ satisfy the following conditions (A1)–(A3):*

(A1) The condition for $n_1, n_2, \ldots, n_k$:

$$n_i \geq \lceil (\Sigma_{j=j_1}^{j_2} m_j + 1)/m_i \rceil, \tag{1}$$

where j_1 and j_2 are given as follows (note that j_1 and j_2 depend on i): If k is even, say $k = 2\ell$, then

$$\begin{cases} j_1 = \max\{i-1, 1\},\ j_2 = \min\{i+s, \ell+1\}, \\ \qquad\qquad \text{if } 1 \leq i \leq \ell, \\ j_1 = \max\{i-s, \ell\},\ j_2 = \min\{i+1, k\}, \\ \qquad\qquad \text{if } \ell+1 \leq i \leq k. \end{cases} \tag{2}$$

On the other hand, if k is odd, say $k = 2\ell - 1$, then

$$\begin{cases} j_1 = \max\{i-1, 1\} \text{ and } j_2 = \min\{i+s, \ell\}, \\ \qquad\qquad \text{if } 1 \leq i \leq \ell - 1, \\ j_1 = i-1 \text{ and } j_2 = i+1, \ \text{if } i = \ell, \\ j_1 = \max\{i-s, \ell\} \text{ and } j_2 = \min\{i+1, k\}, \\ \qquad\qquad \text{if } \ell+1 \leq i \leq k. \end{cases} \tag{3}$$

(A2) The condition for $B_1, B_2, \ldots, B_k$:

$$1 \leq B_i \leq \begin{cases} U_{i-1} + 1, \ \text{if } 1 \leq i \leq s+1, \\ U_{i-1} - U_{i-s-1}, \ \text{if } s+2 \leq i \leq k, \end{cases} \tag{4}$$

and

$$1 \leq B_i \leq \begin{cases} U_{i+s} - U_i, \ \text{if } 1 \leq i \leq k-s-1, \\ U_k - U_i + 1, \ \text{if } k-s \leq i \leq k, \end{cases} \tag{5}$$

where $U_i = \Sigma_{j=1}^{i} |\Psi_j|$ for $i = 1, 2, \ldots, k$. Note that we have $B_1 = B_k = 1$.

(A3) The condition for $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$:

$$1 \leq |\Psi_i| \leq (m_i - 1)B_i + 1 \text{ for } 1 \leq i \leq k, \tag{6}$$

and

$$|\Psi_i| \geq \begin{cases} B_{i-1}, \ \text{if } 2 \leq i \leq \lceil k/2 \rceil, \\ B_{i+1}, \ \text{if } \lfloor k/2 \rfloor + 1 \leq i \leq k-1. \end{cases} \tag{7}$$

The second result gives us the parameters $n_1, n_2, \ldots, n_k$, $B_1, B_2, \ldots, B_k$, and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ for the *optimal* construction that achieves minimum construction complexity and maximum buffer size among the constructions in Theorem 1 for the scenario that $m_i = m_{k-i+1} \geq 2$ for all $1 \leq i \leq \lceil k/2 \rceil$.

Theorem 2 *[13, Theorem 4] Suppose that $1 \leq s \leq k-1$ and $m_i = m_{k-i+1} \geq 2$ for all $1 \leq i \leq \lceil k/2 \rceil$. Then the parameters $n_1, n_2, \ldots, n_k$, $B_1, B_2, \ldots, B_k$, and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ for the optimal construction that achieves minimum construction complexity and maximum buffer size among the constructions in Theorem 1 are given as follows:*

(A1) The optimal choice for $n_1, n_2, \ldots, n_k$:*

$$n_i = \lceil (\Sigma_{j=j_1}^{j_2} m_j + 1)/m_i \rceil, \tag{8}$$

where j_1 and j_2 are given by (2) and (3) (note that j_1 and j_2 depend on i), for $1 \le i \le k$.

(A2*) *The optimal choice for $B_1, B_2, \ldots, B_k$: If $s+1 \le k \le 2s+2$, then $B_1, B_2, \ldots, B_k$ are recursively given by*

$$B_i = B_{k-i+1} = \Sigma_{j=1}^{i-1}((m_j - 1)B_j + 1) + 1 \quad \text{for } 1 \le i \le \lceil k/2 \rceil, \tag{9}$$

where we have adopted the convention that the sum is zero if the upper limit is smaller than the lower limit of a summation. On the other hand, if $k \ge 2s+3$, then $B_1, B_2, \ldots, B_k$ are recursively given by

$$B_i = B_{k-i+1} = \begin{cases} \Sigma_{j=1}^{i-1}((m_j - 1)B_j + 1) + 1, & \text{if } 1 \le i \le s+1, \\ \Sigma_{j=i-s}^{i-1}((m_j - 1)B_j + 1), & \text{if } s+2 \le i \le \lceil k/2 \rceil. \end{cases} \tag{10}$$

Note that we have $B_1 = B_k = 1$.

(A3*) *The optimal choice for $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$:*

$$|\Psi_i| = (m_i - 1)B_i + 1 \text{ for } 1 \le i \le k. \tag{11}$$

Furthermore, the maximum buffer size U_k achieved by the optimal construction is given by

$$U_k = \Sigma_{i=1}^{k}((m_i - 1)B_i + 1). \tag{12}$$

III. Fault-Tolerant Capability of the Optimal Constructions in Theorem 2

In this section, we consider the scenario that each group of FM1's in Figure 2 has the same number of FM1's, say, $m_i = m$ for $1 \le i \le k$, and use Theorem 1 to show that the optimal construction given in Theorem 2 (with $m_i = m$ for $1 \le i \le k$) possesses fault-tolerant capability.

Theorem 3 *Suppose that $1 \le s \le k-1$ and $m_i = m \ge 2$ for $1 \le i \le k$. Then the optimal construction in Theorem 2 (with $m_i = m$ for $1 \le i \le k$) possesses fault-tolerant capability that can tolerate up to f malfunctioning FM1's, i.e., the feedback architecture in Figure 2 can still be operated as an optical PQ but with a smaller buffer size after up to f FM1's fail to function properly, where $f = \min_{1 \le i \le k} \lfloor (m-1)/(n_i - 1) \rfloor$.*

Proof. Note that for the optimal construction in Theorem 2 (with $m_i = m$ for $1 \le i \le k$), the parameters $n_1, n_2, \ldots, n_k$ are given by (8), $B_1, B_2, \ldots, B_k$ are given by (9) and (10), $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ are given by (11), and the maximum buffer size U_k achieved by the optimal construction is given by (12) (with $m_i = m$ in (8)–(12) for $1 \le i \le k$). Since the optimal construction in Theorem 2 is one of the constructions in Theorem 1, the parameters $n_1, n_2, \ldots, n_k$, $B_1, B_2, \ldots, B_k$, and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ satisfy the conditions (A1)–(A3).

Assume that there are f_i broken FM1's in the i^{th} group of FM1's in Figure 2 for $1 \le i \le k$, where $\Sigma_{i=1}^{k} f_i \le f$. Let $f_i' = \max\{f_i, f_{k-i+1}\}$ and let $m_i' = m - f_i'$ for $1 \le i \le k$. Then it is clear that $f_i' = f_{k-i+1}' \le \Sigma_{j=1}^{k} f_j \le f$ and $m_i' = m_{k-i+1}'$ for $1 \le i \le \lceil k/2 \rceil$. For $1 \le i \le k$, we have

$$\begin{aligned} m_i' &= m - f_i' \ge m - f \\ &\ge m - \lfloor (m-1)/(n_i - 1) \rfloor = \lceil m - (m-1)/(n_i - 1) \rceil \\ &\ge \lceil m - (m-1)/2 \rceil = \lceil (m+1)/2 \rceil \ge 2, \end{aligned} \tag{13}$$

where the first inequality follows from $f_i' \le f$, the second inequality follows from $f = \min_{1 \le j \le k} \lfloor (m-1)/(n_j - 1) \rfloor \le \lfloor (m-1)/(n_i - 1) \rfloor$, the third inequality follows from $n_i \ge 3$ (this can be seen from (8), (2), and (3) (with $m_i = m$ for $1 \le i \le k$ in (8), (2), and (3)). and the fourth inequality follows from $m \ge 2$.

Let $B_1', B_2', \ldots, B_k'$ be given by (9) and (10) (with m_i in (9) and (10) replaced by m_i' for $1 \le i \le k$), and let $|\Psi_1'|, |\Psi_2'|, \ldots, |\Psi_k'|$ be given by (11) (with m_i and B_i in (11) replaced by m_i' and $'_i$, respectively, for $1 \le i \le k$). In the following, we show that the parameters $n_1, n_2, \ldots, n_k$, $B_1', B_2', \ldots, B_k'$, and $|\Psi_1'|, |\Psi_2'|, \ldots, |\Psi_k'|$ satisfy the conditions (A1)–(A3) in Theorem 1 (with m_i replaced by m_i' for $1 \le i \le k$), and it then follows from Theorem 1 that the feedback architecture in Figure 2 can still be operated as an optical PQ by using the remaining functioning FM1's.

First we prove that the parameters $n_1, n_2, \ldots, n_k$ satisfy the condition (A1), i.e., $n_1, n_2, \ldots, n_k$ satisfy (1) (with m_i in (1) replaced by m_i' for $1 \le i \le k$) in the condition (A1). To see this, suppose that $1 \le i \le k$. Let j_1 and j_2 be given by (2) and (3) (note that j_1 and j_2 depend on i). Then we have

$$\begin{aligned} &\Sigma_{j=j_1}^{j_2} m_j' + 1 \\ &= \Sigma_{j=j_1}^{j_2} (m - f_j') + 1 = (j_2 - j_1 + 1)m - \Sigma_{j=j_1}^{j_2} f_j' + 1 \\ &= (j_2 - j_1 + 1)(m_i' + f_i') - \Sigma_{j=j_1}^{j_2} f_j' + 1 \\ &\le (n_i - 1)(m_i' + f) - f_i' + 1 \\ &\le (n_i - 1)m_i' + (m-1) - f_i' + 1 \\ &= (n_i - 1)m_i' + m_i' = n_i \cdot m_i', \end{aligned} \tag{14}$$

where the first inequality follows from $n_i = j_2 - j_1 + 2$ in (8) (with $m_j = m$ in (8) for $1 \le j \le k$), $f_i' \le f$, and $\Sigma_{j=j_1}^{j_2} f_j' \ge f_i'$ (as $j_1 \le i \le j_2$), and the second inequality follows from $f = \min_{1 \le j \le k} \lfloor (m-1)/(n_j - 1) \rfloor \le (m-1)/(n_i - 1)$. Thus, we obtain from (14) the desired result that $n_i \ge \lceil (\Sigma_{j=j_1}^{j_2} m_j' + 1)/m_i' \rceil$.

Since we know from (13) that $m_i' = m_{k-i+1}' \ge 2$ for $1 \le i \le \lceil k/2 \rceil$. and the parameters $B_1', B_2', \ldots, B_k'$ and $|\Psi_1'|, |\Psi_2'|, \ldots, |\Psi_k'|$ are determined by $m_1', m_2', \ldots, m_k'$ in the same way that the parameters $B_1, B_2, \ldots, B_k$ and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ are determined by $m_1, m_2, \ldots, m_k$, it then follows that the parameters $B_1', B_2', \ldots, B_k'$ and $|\Psi_1'|, |\Psi_2'|, \ldots, |\Psi_k'|$ satisfy the conditions (A2) and (A3) (just like the parameters $B'1, B_2, \ldots, B_k$ and $|\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ satisfy the conditions (A2) and (A3)).

Finally, since $m_i' = m - f_i' \le m$ for $1 \le i \le k$, it is clear from (9) and (10) that $B_i' \le B_i$ for $1 \le i \le k$. As such, the n_iFM1's with buffer size B_i can be used as n_iFM1's with buffer size B_i' for $1 \le i \le k$. Therefore, we have from Theorem 1 that the feedback architecture in

Figure 2 can still be operated as an optical PQ with buffer size $U_k' = \Sigma_{i=1}^k ((m_i'-1)B_i'+1))$. As we can see from $m_i' \le m$ and $B_i' \le B_i$ for $1 \le i \le k$ that

$$\begin{aligned} U_k' &= \Sigma_{i=1}^k ((m_i'-1)B_i'+1)) \\ &\le \Sigma_{i=1}^k ((m-1)B_i+1)) = U_k, \end{aligned}$$

we know that the buffer size U_k' achieved by using the remaining functioning FM1's is less than or equal to the buffer size U_k achieved when no FM1's are broken. ■

Remark 4 *(i) The result in [13, Theorem 10] is a special case of our result in Theorem 3 when each FM1 in Figure 2 has the same number of arrival links.*

(ii) The result in Theorem 3 can be used to provide quality of service (QoS). For example, to guarantee a fault-tolerant capability of f^* broken FM1's, the parameters m and $n_1, n_2, \ldots, n_k$ have to be chosen such that $\min_{1\le i\le k}\lfloor (m-1)/(n_i-1)\rfloor \ge f^*$.

IV. Conclusion

In this paper, we addressed the fault-tolerant capability of the optimal constructions of optical PQs in [13] by using the feedback architecture in Figure 2. We considered the scenario that each group of FM1's has the same number of FM1's, and showed that the optimal constructions in [13] possess fault-tolerant capability which can be expressed in terms of the number of FM1's in each group and the numbers of arrival links of the FM1's in the k groups in Figure 2. Such a result can be used in the design of the construcrions to provide QoS that guarantees a certain level of fault-tolerant capability.

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