A Min-Max Approximation-Based SDP Approach to Robust Vehicle Location Estimation in VANETS

Tan-Jan Ho  
Dept. of Electrical Engineering  
Chung Yuan Christian University  
Taoyuan City, Taiwan, R.O.C.  
Email: tjho@cycu.edu.tw

Hua-Yu Luo  
Dept. of Electrical Engineering  
Chung Yuan Christian University  
Taoyuan City, Taiwan, R.O.C.  
Email: keyis0935128641@gmail.com

Abstract—In this paper, we consider robust vehicle localization in a vehicular ad-hoc network (VANET) under changing vehicle velocities and probabilistically mixed line-of-sight (LOS)/non-LOS (NLOS) conditions, where NLOS error statistics and occurrence probabilities are unknown. To tackle the problem, we develop two localization algorithms based on a proposed min-max approximation (MMA)-based semidefinite programming (SDP) method and a proposed advanced measurement preprocessing (AMP) scheme. The AMP method is used for reducing the NLOS influence on raw range measurements received by multiple base stations. Accordingly, we have AMP-MMA and AMP-Joint MMA algorithms. The latter can perform better than the former because it additionally incorporates a neighbor-assisting localization scheme. Simulations show that the two proposed location estimation algorithms perform satisfactorily and outperform the MMA and some tracking algorithms in the literature.

Keywords—Vehicular Ad-Hoc Network (VANET), Robust Vehicle Localization, Semidefinite Programming (SDP), Advanced Measurement Preprocessing (AMP), AMP-Based Min-Max Approximation (AMP-MMA), AMP-Joint MMA.

I. INTRODUCTION

A vehicular ad-hoc network (VANET) can be regarded as a wireless communication network consisting of vehicles and roadside infrastructures (e.g., base stations, speed camera or relay nodes). The vehicle to infrastructure (V2I) and vehicle to vehicle (V2V) methods are two main communication schemes to be employed in a VANET. Moreover, vehicles can communicate with each other by sending and accessing information via Internet of Things techniques (e.g., [1],[2]). For instance, the average speed of a vehicle in a VANET is accessible since information can be relayed among cars. Thus, the positioning performance of a vehicle can be improved due to the joint assistance of other vehicles in the same network [3].

In this paper, we consider robust vehicle localization in a VANET. Although a global positioning system (GPS) can be used for target tracking, many GPS denied environments exist because signal transmissions in urban districts are highly likely to be blocked by massive constructions, pedestrians and vehicles. Thus, we utilize cellular networks for tackling the location estimation problem. Because non-line-of-sight (NLOS) signal propagations arise frequently and may follow an unknown probability distribution, range measurements received by multiple BSs from a target vehicle in a VANET are under probabilistically mixed line-of-sight (LOS)/non-LOS (NLOS) conditions, where the NLOS error statistics and occurrence probabilities (NOPs) are unknown. It is a huge challenge to meet the recent stringent localization accuracy requirement of the US Federal Communications Commission (FCC) reported in [4], namely, localization root-mean-squares errors (RMSEs) of no more than 50 m for 80% of 30 s.

Numerous methods (e.g., [5]-[10]) may be employed for tackling aforementioned vehicle tracking issues. But, they may not achieve desirable performance under large NOPs and varying vehicle velocities. Specifically, the methods in [5], [10] are constraint optimization forms using second-order cone relaxation based on various assumptions. Since they do not take NLOS statistics and NOPs into account, their performance may not be desirable. The algorithms in [6],[7] can only track a target well under small NOPs and constant target velocities. Though the methods in [8],[9] can perform satisfactorily regardless of NOPs when the target moves with a constant velocity, their positioning performance may degrade under varying vehicle speeds. Moreover, a careful selection of a state-space model-based system framework in [6]-[9] is required to yield desirable performance.

To avoid the drawbacks in the aforementioned observations, we are motivated to employ robust optimization techniques (e.g., [10],[11] and references therein) without the requirement of a state-space system model to develop two new convex optimization-based algorithms for vehicle tracking. In summary, the main contributions of this paper are given as follows.

- We introduce a min-max approximation (MMA)-based method for developing proposed tracking algorithms.
- Based on available vehicles’ velocities in a VANET [12], we propose an advanced measurement preprocessing (AMP) method for dynamically trimming NLOS-corrupted raw range measurements received by multiple base stations.
- We propose an AMP-MMA algorithm by incorporating the AMP into the MMA, and then an AMP-Joint MMA algorithm by incorporating a proposed neighbor-assisting localization scheme into the AMP-MMA.
• We demonstrate that the AMP-MMA and AMP-Joint MMA perform satisfactorily and outperform the MMA and some tracking algorithms significantly. Additionally, the latter can perform better than the former.

This paper is organized as follows. Sec. II describes the problem setting. Sec. III presents the development of the proposed methods. Sec. IV shows simulation results that the proposed AMP-MMA and AMP-Joint MMA can give satisfactory performance. Sec. V gives a summary.

II. PROBLEM STATEMENT

The vehicle localization setup is given as follows. The vehicle position is denoted as \( \mathbf{x}_k = [x_k \ y_k]^T \) where the subscript \( k \) stands for the discrete-time index; \( x_k \) and \( y_k \) denote the respective position in the x-y plane. The location of the \( i \)th base station (denoted by BS\(_i\)), \( i = 1 \ldots M \), is given by \( \mathbf{x}_{BS_i} = [x_{BS_i} \ y_{BS_i}]^T \) where \( x_{BS_i} \) and \( y_{BS_i} \) denote the respective BS\(_i\) position in the x-y plane. The distance between the vehicle and BS\(_i\) at time \( k \) is given by

\[
d_{k,i} = \sqrt{(x_k - x_{BS_i})^2 + (y_k - y_{BS_i})^2}.
\]

Then, (1) can be written as

\[
d_{k,i}^2 = ||\mathbf{x}_k - \mathbf{x}_{BS_i}||^2 = \mathbf{x}_k^T \cdot \mathbf{x}_k - 2x_k \cdot x_{BS_i} + x_{BS_i}^T \cdot \mathbf{x}_{BS_i}.
\]

The range measurement received by BS\(_i\) at time \( k \) is given by

\[
z_{k,i} = d_{k,i} + \delta_{k,i},
\]

where the measurement error \( \delta_{k,i} \) is caused by LOS or NLOS noise, and thus given by

\[
\delta_{k,i} = \begin{cases} 
\sigma_L \cdot \omega_{k,i} & \text{if LOS noise occurs} \\
\sigma_{NL} \cdot \omega_{k,i} & \text{if NLOS noise occurs}
\end{cases}
\]

where \( \sigma_L \) denotes the known LOS noise standard deviation; \( \sigma_{NL} \) denotes a zero-mean Gaussian random variable with unit variance; \( N_{k,i} \) stands for an unknown NLOS bias; and \( \sigma_{NL} \) denotes an unknown NLOS noise standard deviation.

Our objective is to estimate the target vehicle location \( \mathbf{x}_k \) based on the measurements \( z_{k,i} \) in (3) received by BS\(_i\), \( i = 1 \ldots M \). We are motivated by robust optimization techniques in [10],[11] to cast the problem into a min-max form as

\[
\min \max_{\mathbf{x}_k} \left| \frac{1}{k} \sum_{i=1}^{M} \left( z_{k,i}^2 - d_{k,i}^2 \right) \right|.
\]

The estimated target vehicle position is given by

\[
\hat{\mathbf{x}}_k = \arg \min_{\mathbf{x}_k} \max_{i=1 \ldots M} \left| \frac{1}{k} \sum_{i=1}^{M} \left( z_{k,i}^2 - d_{k,i}^2 \right) \right|.
\]

Since (5) is non-convex, we introduce a min-max approximation (MMA)-based semi-definite programming (SDP) approach to relax (5) into a SDP problem. Because the MMA-based algorithm in [11] only considers the LOS condition and restricts the search region of several parameters in a small box, it cannot be utilized in our mixed LOS/NLOS setting. Our approach utilizes actual target vehicle velocities at the x- and y-axis (denoted by \( v_{x,\text{Target}} \) and \( v_{y,\text{Target}} \), respectively), which can be obtained from a VANET, for enhancing the accuracy of vehicle location computing.

III. MAIN RESULTS

In the following, we present the proposed algorithms.

A. Proposed Min-Max Approximation (MMA)

We relax (5) into a SDP problem by introducing the following constraint

\[
x_{k,i} = x_{k,i}^T \cdot x_k.
\]

Based on (3) and (7), we have

\[
\frac{z_{k,i}^2 - d_{k,i}^2}{v_{x,\text{Target}} v_{y,\text{Target}}} \approx x_{k,i}^T \cdot \mathbf{x}_{\text{Target}} - 2x_k \cdot x_{\text{Target}} + x_{\text{Target}}^T \cdot x_{\text{Target}}.
\]

Based on (8), (5) can be approximated as

\[
\min \max_{\mathbf{x}_k} \left| x_{k,i}^T \cdot \mathbf{x}_{\text{Target}} - 2x_k \cdot x_{\text{Target}} + x_{\text{Target}}^T \cdot x_{\text{Target}} \right|.
\]

Because the vehicle largest allowable velocity and acceleration are bounded, the vehicle location change \( |x_k - \hat{x}_{k-1}| \) at each time \( k \) is restricted to a box, i.e.,

\[
|x_k - \hat{x}_{k-1}| \leq \alpha_{\text{Max}} \cdot T + v_{x,\text{Target}} \cdot T + \frac{1}{2} a_{\text{Target}} \cdot T^2
\]

\[
|x_k - \hat{x}_{k-1}| \leq \alpha_{\text{Max}} \cdot T + v_{y,\text{Target}} \cdot T + \frac{1}{2} a_{\text{Target}} \cdot T^2
\]

where \( \hat{x}_{k-1} \) is the estimated vehicle position at time \( k-1 \); \( \alpha_{\text{Max}} \) denotes the vehicle’s largest allowable velocity; \( T \) denotes the sampling period; and the target’s accelerations can be obtained by computing

\[
a_{x,\text{Target}} = \frac{\text{Target} \cdot x_{k-1} - x_k}{T}, \quad a_{y,\text{Target}} = \frac{\text{Target} \cdot y_{k-1} - y_k}{T}.
\]

Eqns. (7),(9) and (10) yield the following min-max approximation (MMA), which can be solved by the SDP

\[
\min \theta_k \quad \text{s.t.} \quad -\theta_k \leq x_{k,i}^T \cdot \mathbf{x}_{\text{Target}} - 2x_k \cdot x_{\text{Target}} + x_{\text{Target}}^T \cdot x_{\text{Target}} \leq \theta_k.
\]

B. Proposed Advanced Measurement Preprocessing (AMP) Based MMA (AMP-MMA) Algorithm

Since raw range measurements received by BSs may contain contaminated NLOS outliers, we propose an AMP method for
dynamically clipping raw range measurements to reduce NLOS effects on positioning performance. Like [6]-[9], we select a reasonably small sampling period \( T \) and start with a suitable initial vehicle location \( x_0 = \begin{bmatrix} t_0^{\text{pred}} & y_0^{\text{pred}} \end{bmatrix}^T \). Based on the Newton’s second law of motion, the predicted vehicle position \((\hat{x}_k^{\text{Pred}}, \hat{y}_k^{\text{Pred}})\) at time \( k \) is given by

\[
\begin{align*}
\hat{x}_k^{\text{Pred}} &= \hat{x}_{k-1}^{\text{Pred}} + v_{x,k-1}^{\text{Target}} \cdot T + \frac{1}{2} a_{x,k-1}^{\text{Target}} T^2 \\
\hat{y}_k^{\text{Pred}} &= \hat{y}_{k-1}^{\text{Pred}} + v_{y,k-1}^{\text{Target}} \cdot T + \frac{1}{2} a_{y,k-1}^{\text{Target}} T^2
\end{align*}
\] (13)

Then, the predicted range measurements \( z_{k,j}^{\text{pred}} \) w.r.t. BS\( i \), \( i = 1, \ldots, M \) at time \( k \) are given by

\[
z_{k,i}^{\text{pred}} = \sqrt{(\hat{x}_k^{\text{Pred}} - x_{i,k})^2 + (\hat{y}_k^{\text{Pred}} - y_{i,k})^2}.
\] (14)

The value \( |z_{k,i} - z_{k,i}^{\text{pred}}| \) can be thought of as an estimated noise standard deviation. If it is greater than the LOS standard deviation, then we clip the received measurement which may be highly contaminated with NLOS outliers. Thus, we have the following processed measurements

\[
z_{k,i} = \begin{cases} z_{k,i}^{\text{pred}} & \text{if } |z_{k,i} - z_{k,i}^{\text{pred}}| \leq \sigma_L \\ z_{k,i} & \text{otherwise} \end{cases}
\] (15)

We have \( \psi_{k,i}(z_{k,i}^{\text{pred}}, \hat{x}_{k,i}, \hat{y}_{k,i}, x_{i,k}, y_{i,k}, x_{BS_i}, y_{BS_i}) \) with \( z_{k,i}^{\text{pred}} \) in (12) replaced by \( z_{k,i} \) in (15) to yield the proposed AMP-based MMA location estimator denoted as AMP-MMA as follows:

---Proposed AMP-MMA---

**Step 1.** Compute Eqns.(11), (13)-(15)

**Step 2.** Compute

\[
\min_{\theta_k} \quad \psi_{k,i}(z_{k,i}, \hat{x}_{k,i}, \hat{y}_{k,i}, x_{i,k}, y_{i,k}, x_{BS_i}, y_{BS_i}) - \theta_k,
\]

Eq.(10),

\[
\begin{bmatrix} 1 & \hat{x}_k^T & x_{i,k} \end{bmatrix} \geq 0.
\]

**B. Proposed AMP-Based Joint MMA (AMP-Joint MMA) Algorithm**

We consider that a vehicle near the target vehicle in a VANET is utilized as a mobile station (MS) for assisting in locating the target as shown in Fig.1. The MS position also needs to be estimated. This is different from the method in [13], which utilizes map information and GPS data. Since the locations of the target vehicle and the MS need to be estimated, we can incorporate additional constraints into the MMA to carry out the location estimation.

To begin with, let \( y_k = \begin{bmatrix} M \hat{x}_k^M \end{bmatrix}^T \) denote the MS position.

The range measurement between the MS and BS\( i \) is given by

\[
z_{k,i}^M = d_{k,i}^M + \delta_{k,i}^M
\] (17)

where \( d_{k,i}^M \) is the true range between the MS and BS\( i \), and \( \delta_{k,i}^M \) is the measurement error. As MS velocities \( v_{x,k,i-1}^M, v_{y,k,i-1}^M \) are available, we can get \( z_{k,i}^{M-pred}, \delta_{k,i}^M \) in a manner similar to (11). The predicted MS position \((\hat{x}_{k,i}^{M-pred}, \hat{y}_{k,i}^{M-pred})\) can be obtained in a manner similar to (13). Hence, the predicted range measurements \( z_{k,i}^{M-pred} \) w.r.t. BS\( i \) with \( i = 1, \ldots, M \) can be obtained in a way like (14). Like (15), we have processed measurement \( z_{k,i}^{M-pred} \). Like (7), we use the constraint

\[
y_{k,i} = y_k^T \cdot \hat{y}_{k,i}.
\] (18)

Like (6) and (8), we have

\[
\psi_{k,i}(z_{k,i}^{M-pred}, \hat{x}_{k,i}^{M-pred}, \hat{y}_{k,i}^{M-pred}, x_{i,k}^M, y_{i,k}^M, x_{BS_i}^M, y_{BS_i}^M) = (z_{k,i}^{M-pred})^2 - (y_{k,i} - y_{BS_i})^2.
\]

Like (10), we have

\[
\begin{bmatrix} 1 & \hat{x}_k^{M-pred}^T & x_{i,k}^M \end{bmatrix} \geq 0.
\]

**Proposed AMP-Joint MMA---**

**Step 1.** Compute Eqns.(11),(13)-(15), \( \delta_{k,i}^{M-pred}, \delta_{k,i}^M \)

**Step 2.** Compute

\[
\begin{bmatrix} 1 & \hat{x}_k^{M-pred}^T & x_{i,k}^M \end{bmatrix} \geq 0.
\]

---Proposed AMP-Joint MMA---

**Step 1.** Compute Eqns.(11),(13)-(15), \( \delta_{k,i}^{M-pred}, \delta_{k,i}^M \)

**Step 2.** Compute

\[
\begin{bmatrix} 1 & \hat{x}_k^{M-pred}^T & x_{i,k}^M \end{bmatrix} \geq 0.
\]
The performances of our proposed algorithms, the SCOP-\ell_2 [5], the RIMM in [6] and the M-REKF in [8] are compared. The parameter values for the SCOP-\ell_2 follow those in [5]. The algorithm in [10] is not suitable for the problem setting herein because it requires several additional parameters used for optimization. Thus, no results of the algorithm in [10] can be obtained for performance comparison.

Fig. 3 displays the performance comparisons of the proposed MMA, proposed AMP-MMA, the SCOP-\ell_2, the RIMM and the M-REKF under Speed Scenario A and NOP 0.75. Regardless of the NOP, the proposed AMP-MMA and the M-REKF perform well and significantly outperform other competitors under the constant velocity. This is mainly because they clip received measurements to reduce the influence of NLOS outliers prior to carrying out location estimation.

Fig. 4 shows performance comparisons of those methods listed in Fig. 3 under Speed Scenario B and NOP 0.75. The proposed AMP-MMA is superior to other competitors. This is mainly because the AMP trims NLOS outliers from received measurements under changing vehicle velocities. It is noted the SCOP-\ell_2, RIMM, and M-REKF perform poorly under varying vehicle velocities. The proposed AMP-MMA can meet the FCC positioning requirements as long as the initial position RMSEs are no more than 50m.

Fig. 5 shows that the proposed AMP-Joint MMA performs better than the proposed AMP-MMA under Speed Scenario A and NOP 0.75 due to the assistance of joint localization, which utilizes estimated neighbor’s position.

Fig. 6 displays the performance comparisons of the proposed algorithms under Speed Scenario B and NOP 0.75. Despite varying vehicle velocities, the proposed AMP-Joint MMA outperforms the proposed AMP-MMA due to the neighbor-assisting localization scheme.

Like the AMP-MMA, the AMP-Joint MMA can meet the FCC positioning requirement as long as the initial poisoning the initial position RMSEs are no more than 50m.

The computational times of the AMP-MMA and AMP-Joint MMA per time step iteration are approximately 8.7ms and 9.1 ms, respectively. The result substantiates that the AMP-MMA performs more efficiently than the AMP-Joint MMA. This is consistent with Remark 1.

\[ \min_{\hat{x}_{k,i}, \hat{y}_{k,i}} \sum_{k} \sum_{i} \left( \left( \hat{x}_{k,i} - x_{k,i} \right)^2 + \left( \hat{y}_{k,i} - y_{k,i} \right)^2 \right) \]

\[ \text{s.t.} \quad -\theta_k \leq \psi_{k,i}(\hat{x}_{k,i}, \hat{y}_{k,i}) \leq \theta_k, \]

\[ -\theta_k \leq \psi_{k,i}(\hat{x}_{k,i}, \hat{y}_{k,i}) \leq \theta_k, \]

\[ -\theta_k \leq \psi_{k,i}(\hat{x}_{k,i}, \hat{y}_{k,i}) \leq \theta_k, \]

\[ \left[ \begin{array}{c} x_k \\ y_k \end{array} \right] \geq 0, \quad \left[ \begin{array}{c} y_k \\ y_k \end{array} \right] \geq 0. \]

**Remark 1:** As shown in (16) and (25), the proposed AMP-Joint MMA extends the algorithmic functions and structures of the proposed AMP-MMA. Thus, the computational load of the AMP-Joint MMA is higher than that of the AMP-MMA.

### IV. SIMULATIONS

In this section, we illustrate that the proposed algorithms can achieve satisfactory tracking performance under changing environment conditions and varying vehicle velocities in comparison with some other algorithms in the literature.

#### A. Environment Setting

We consider a VANET with five BSs (i.e., \( M = 5 \)). The BS locations are given by \( B_5 = (500,500) \), \( B_2 = (500,2500) \), \( B_3 = (2500,500) \), \( B_4 = (2500,2500) \), and \( B_5 = (1500,1000) \), respectively. The initial locations of the target vehicle and MS are \( x_0 = [1036 \ m \ 1036 \ m]^T \) and \( y_0 = [1036 \ m \ 1036 \ m]^T \), respectively. Their initial predicted locations are randomly generated as \( x_0 = [1036 + 30 \times \text{randn}] \) and \( y_0 = [1036 + 30 \times \text{randn}] \), with \( \text{randn} \) denoting a number generated from a zero-mean Gaussian distribution with unit variance. The sampling time period is \( T = 0.2 s \). Let \( \sigma_L = 150 \ m \) and \( \sigma_{NL} = 150 \ m \) in (4). The NLOS bias model is a shifted Gaussian distribution with mean 1000 m and standard deviation 300 m (e.g., [6]-[9]). We consider the target vehicle and MS velocities as follows:

**Speed Scenario A:**

The constant velocities of the target and MS are given by \( (v_{x,k}, v_{y,k}) = (4, 4) \) m/s and \( (v_{Mx,k}, v_{My,k}) = (4, 4) \) m/s.

**Speed Scenario B:** Time varying x- and y-axis velocities of the target and MS are shown in Fig. 2.

#### B. Performance Comparison

We utilize the MOSEK optimization toolbox of Matlab R2020b on a R9-3900X PC with 32G RAM for performing 100 Monte Carlo simulation runs for each algorithm. Each simulation run is carried out 1000 time steps. The location root-mean-square error (RMSE) at the kth time step is given by

\[ \sqrt{\frac{1}{100} \sum_{i=1}^{100} \left( x_k - \hat{x}_k \right)^2 + \left( y_k - \hat{y}_k \right)^2 } \]

where \( x_k, y_k \) and \( \hat{x}_k, \hat{y}_k \) are the target true and estimated positions for the i\textsuperscript{th} run.

Fig. 2 Varying x- and y-axis velocities of the target (shown in the first two graphs) and those of the MS (displayed in the third graph)
V. SUMMARY

In this paper, we have presented the AMP-MMA and AMP-Joint MMA algorithms for robust vehicle tracking in a VANET under challenging environment conditions and varying vehicle velocities. The proposed advanced measurement preprocessing (AMP) scheme utilizes available vehicle velocities for alleviating the impact of NLOS errors on received measurements. So, it has enabled the proposed tracking algorithms to yield desirable positioning performance. The AMP-Joint MMA is obtained by integrating a proposed neighbor-aided localization scheme into the AMP-MMA for more effective target tracking. Simulations have demonstrated that the AMP-MMA and the AMP-Joint MMA can indeed yield satisfactory positioning performance and outperform some tracking algorithms in the literature. In addition, the AMP-Joint MMA can perform better than the AMP-MMA.

ACKNOWLEDGMENT

This work was supported in part by the National Science and Technology Council of Taiwan under the grant no. MOST-111-2221-033-048.

REFERENCES