

A Game Theory Based Rational Mining Strategy in Blockchains With Multiple Rational Miners

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Abstract—Previous researches have shown that a miner using selfish mining strategy is not necessarily profitable if the selfish mining rate is not large enough. In such situation, a miner will choose honest mining strategy rather in order to earn more rewards. Such miners are called rational miners. In this paper, we discuss mining strategies in the blockchains with one honest and two independent rational miners. The rewards earned by different miners using different mining strategy combinations are calculated by the analytical models proposed in previous works. Payoff matrices are constructed and the Nash equilibria are then found by comparing the earned rewards. Numerical results show that the miners have their own dominant strategies in most situations. However, when the mining rates of the two rational miners are both between 0.22 and 0.25, there is no dominant strategy such that the miners shall guess which strategy will be used by the other miner. We proposed a game theory based rational mining strategy that chooses the mining strategy according to a probability distribution. Rewards earned by the proposed strategy are shown to be guaranteed larger than or equal to those earned by honest mining strategy.

Index Terms—Blockchain, mining, game theory

I. INTRODUCTION

Blockchain is a distributed ledger in which transactions are securely stored in the consecutive blocks [1]. The blocks are chained one by one by storing the hash value of previous block in the current one such that the transactions are immutable. Mining in a blockchain is a process to find a nonce which is attached into the block resulting in that hash value of the block begins with a predefined number of zero bits [1]. The first node finding the nonce is entitled to append the block to blockchain and to earn rewards. Nodes competing to be the first one finding the nonce are called the *miners*.

When a miner mined the next block, all the other miners are notified to validate the block [1]. If the block is valid, the miner who mined the block earns rewards. Generally, the rewards earned by a miner is proportional to his mining rate; i.e. the number of nonces he can try to find the valid hash value of a block. However, a mining strategy called *selfish mining* [2] enables a miner to be *profitable*; that is, he can earn more than he is entitled to.

Main idea of selfish mining is to hide the mined block and waste others' mining efforts. A miner using selfish mining strategy is called a *selfish miner* while the miner who announces the mined block immediately is called an *honest*

miner. When a selfish miner mined a new block, he did not announce the block and then starts to mine the next one while others are still finding the nonce of the previous block. In [2], the authors show that the miner is profitable if the fraction of his mining rate is larger than 25%. Furthermore, if a miner with less than 25% of all mining rates employs selfish mining strategy, the fractions of rewards he earns is less than 25%.

If a miner is rational, he may choose honest rather than selfish mining strategy in order to earn more rewards if his mining rate is not large enough. Such miners are called the *rational miners*. In a blockchain with a single rational miner and all others are honest miners, it has been shown that the miner can be profitable if the fraction of his mining rate is larger than 25% [2]. Since selfish mining strategy enables a miner to be profitable, there will be multiple independent miners with sufficient mining rates employing the selfish mining strategy. In this paper, we consider the blockchain with two independent rational miners.

A number of researches have been proposed to address the earned rewards analysis problem in a blockchain with multiple rational miners [3]–[5]. The authors use simulation tools to study the behaviors of rational miners. The previous researches all assume that the miners choose the best strategy given the other miner's strategy. However, the miners are all independent without knowing others' decisions. In this paper, we apply the game theory based concepts [6] to find the best strategies of rational miners.

We first employ two accurate analytical models proposed in previous researches [2], [7] to calculate the rewards earned by different miners under different combinations of strategies. Payoff matrices which are widely used in game theory are constructed based on the mining rates of rational miners. Nash equilibria are found in the payoff matrices. A game theory based rational mining strategy is proposed based on the strategies found in the payoff matrices. If there exists a single Nash equilibrium, a dominant strategy exists and the behaviors of the rational miners is easy to be determined. However, there will be two Nash equilibria in some situations which make the decision difficult to be predicted. To solve the problem, we proposed a mixed strategy to choose the mining strategy by a probability distribution. Numerical results show that the proposed strategy is guaranteed to yield more earned rewards

than the honest mining strategy.

The rest of this paper is organized as follows. The earned rewards are calculated in the next Section. Payoff matrices are constructed and Nash equilibria are found in Section III. A game theory based rational mining strategy are proposed and discussed in Section IV. The numerical results are shown and discussed in Section V. Finally, some concluding remarks are given.

II. EXPECTED EARNED REWARDS

Accurate analytical models for rewards earned by the honest and one or two selfish miners in a blockchain with selfish miners have been proposed in [2], [7] respectively. The rewards earned by the selfish and honest miners in the blockchain with a single selfish miner can be calculated by the analytical model proposed in [2]. The analytical model proposed in [7] studied the earned rewards in a blockchain with two selfish miners accurately. Both analytical models are employed to calculate the earned rewards under different combinations of mining strategies.

Note that we consider a blockchain with one honest and two rational miners. The two rational miners may choose honest or selfish mining strategy according to the rewards they expect to earn. We named the honest miner *Henry* and the other two rational miners *Alice* and *Bob* for the ease of understanding. Assume that the numbers of blocks mined by Henry, Alice and Bob in a time unit follow the Poisson process with mean values r_h , r_a , and r_b respectively. Without loss of generality, we let sum of mining rates equal to 1. That is, $r_h + r_a + r_b = 1$. Therefore, the value of mining rate of a miner equals to the fraction of his mining rate.

We denote H and S as the honest and selfish mining strategies respectively. Four different combinations of strategies employed by Alice and Bob are $\{HH, HS, SH, SS\}$ where strategy XY represents that the strategies employed by Alice and Bob are X and Y respectively. Let R_a^{XY} and R_b^{XY} denote the fractions of rewards earned by Alice and Bob under mining strategy combination XY . The analytical models to calculate the values of R_a^{XY} and R_b^{XY} are employed as follows.

A. Both Alice and Bob are Honest

Since Alice, Bob, and Henry are all honest miners, the fractions of rewards earned by Alice and Bob equal to r_a and r_b respectively [1]. That is, $R_a^{HH} = r_a$ and $R_b^{HH} = r_b$.

B. Alice is Selfish and Bob is Honest

In this situation, only Alice uses selfish mining strategy. The fraction of rewards earned by Alice is as follows [2].

$$R_a^{SH} = \frac{r_a(1-r_a)^2[4r_a + \frac{1}{2}(1-2r_a)] - r_a^3}{1-r_a[1+(2-r_a)r_a]} \quad (1)$$

Since Henry and Bob are both honest miners, they share the remainder rewards in proportional to their mining rates. That is,

$$R_b^{SH} = \frac{r_b}{r_h+r_b} \times (1-R_a^{SH}) = \frac{r_b}{1-r_a} \times (1-R_a^{SH}) \quad (2)$$

TABLE I: Payoff Matrix

Rewards (Alice, Bob)		Bob's Strategy	
		Honest	Selfish
Alice's Strategy	Honest	R_a^{HH}, R_b^{HH}	R_a^{HS}, R_b^{HS}
	Selfish	R_a^{SH}, R_b^{SH}	R_a^{SS}, R_b^{SS}

C. Alice is Honest and Bob is Selfish

Same as the previous discussion, Bob is the only selfish miner and Alice shares the remainder rewards with Henry. That is,

$$R_b^{HS} = \frac{r_b(1-r_b)^2[4r_b + \frac{1}{2}(1-2r_b)] - r_b^3}{1-r_b[1+(2-r_b)r_b]} \quad (3)$$

and

$$R_a^{HS} = \frac{r_a}{r_h+r_a} \times (1-R_b^{HS}) = \frac{r_a}{1-r_b} \times (1-R_b^{HS}) \quad (4)$$

D. Both Alice and Bob are Selfish

An accurate analytical model for rewards earned by two selfish miners has been proposed in [7]. Given the mining rates of Henry r_h , Alice r_a , and Bob r_b , a finite state machine is proposed and the earned rewards can be accurately and efficiently calculated via closed-form expressions. The fractions of earned rewards R_a^{SS} and R_b^{SS} can be calculated by equations (5) as follows.

$$\begin{aligned} R_a^{SS} &= [2r_a^2(1+r_h) + (r_a+r_b)r_ar_h/2 + r_ar_br_h \\ &\quad + 4r_a^2r_b(1+r_h) + 2r_ar_br_h^2/3]/R^{SS} \\ R_b^{SS} &= [2r_b^2(1+r_h) + (r_a+r_b)r_br_h/2 + r_ar_br_h \\ &\quad + 4r_b^2r_a(1+r_h) + 2r_ar_br_h^2/3]/R^{SS} \\ R_h^{SS} &= [3r_ar_h^2 + 8r_ar_b/3 + r_h + 3r_br_h^2 \\ &\quad + 4r_ar_br_h/3]/R^{SS} \end{aligned} \quad (5)$$

where $R^{SS} = R_h^{SS} + R_a^{SS} + R_b^{SS}$.

III. PAYOFF MATRICES AND NASH EQUILIBRIA

Payoff matrices are widely used in game theory to find Nash equilibrium/equilibria [6]. After obtaining the values of the earned rewards, we construct payoff matrices as shown in Table I. The rewards earned by Alice and Bob are shown in each cell respectively. Values of R_a^{HH} and R_a^{SH} are compared in order to determine which strategy of Alice is better given that Bob is using honest strategy. We also compare the values of R_a^{HS} and R_a^{SS} , R_b^{HH} and R_b^{HS} , and R_b^{SH} and R_b^{SS} given one miner has determined her/his mining strategy.

The payoff matrices with mining rates between 0.1 and 0.4 are shown in appendix A. In the matrices shown in Tables II, III, IV, V, and VI, the best strategies are in bold and the Nash equilibria are shown in gray background. Based on the values and Nash equilibrium/equilibria in each payoff matrix, the payoff matrices can be classified into three types. For the ease of understanding, we investigate the earned rewards from the point of view of Alice.

TABLE II: Two miners both have dominant strategies

Rewards (Alice, Bob)		Bob $r_b = 0.1$	
		Honest	Selfish
$r_a = 0.2$	Honest	0.200,0.100	0.206,0.072
	Selfish	0.182,0.102	0.199,0.076
Rewards (Alice, Bob)		Bob $r_b = 0.4$	
		Honest	Selfish
$r_a = 0.3$	Honest	0.300,0.400	0.237, 0.526
	Selfish	0.327,0.385	0.262,0.555

TABLE III: Only one miner has dominant strategy

Rewards (Alice, Bob)		Bob $r_b = 0.21$	
		Honest	Selfish
$r_a = 0.24$	Honest	0.240,0.210	0.244,0.195
	Selfish	0.236, 0.211	0.262,0.210
Rewards (Alice, Bob)		Bob $r_b = 0.27$	
		Honest	Selfish
$r_a = 0.23$	Honest	0.230,0.270	0.227, 0.280
	Selfish	0.222,0.273	0.234,0.320

A. Both miners have dominant strategy

As shown in Tables V and VI, when the mining rate of Alice is less than 0.22 or larger than 0.25, Alice has a dominant strategy. When mining rate of Alice is less than 0.22, she shall use honest mining strategy no matter which strategy is used by Bob. She shall use selfish mining strategy when her mining rate is larger than 0.25. Table II shows the examples that two miners both have their dominant strategies

B. Only one miner has dominant strategy

Table VI shows more detailed payoff matrices when the mining rates are between 0.2 and 0.3. When the mining rate of Alice is between 0.22 and 0.25, she shall choose the better strategy according to Bob's mining strategy. If Bob uses honest mining strategy, Alice shall choose honest mining strategy too in order to earn more rewards. However, if Bob uses selfish mining strategy, Alice shall employ the selfish mining strategy.

We have shown that when Bob's mining rate is less than 0.22 or larger than 0.25, he has a dominant strategy. Therefore, Alice is able to choose her strategy since the Bob's decision is rational and predictable. In the payoff matrices, one Nash equilibrium exists too. Table III shows examples that only one miner has dominant strategy. Thus, the other miner has dominant strategy too.

C. No miner has dominant strategy

If the mining rates of Alice and Bob are both between 0.22 and 0.25, the situation becomes complicated. Both miners have no dominant strategy and the best strategies of both miners depend on each other. Table IV shows the example that no miner has dominant strategy. We found that two Nash equilibria exist in the payoff matrices when both miners choose the same strategy.

Since Alice and Bob are independent and have no knowledge about the other's decision, predicting the other's decision

TABLE IV: No miner has dominant strategy

Rewards (Alice, Bob)		Bob $r_b = 0.24$	
		Honest	Selfish
$r_a = 0.23$	Honest	0.230,0.240	0.231,0.236
	Selfish	0.222,0.243	0.242,0.261

according to a probability distribution is a solution. In a payoff matrix with two Nash equilibria, a mixed strategy can be applied to solve the long-term behavior of each miner [6]. Main idea of the mixed strategy is to make the other miner earn *indifferent* rewards no matter which strategy the other miner uses.

Suppose that Alice selects honest strategy with probability p where $0 \leq p \leq 1$; while she selects selfish mining strategy with probability $1-p$. Same as Bob, he selects honest and selfish mining strategies with probability q and $1-q$ respectively. A mixed strategy hopes to find a probability distribution such that Bob earns *indifferent* rewards no matter which strategy is employed by Bob.

If Bob uses honest mining strategy, the reward earned by Bob equals to $p \times R_b^{HH} + (1-p) \times R_b^{SH}$. If selfish mining strategy is employed by Bob, the reward earned by Bob equals to $p \times R_b^{HS} + (1-p) \times R_b^{SS}$. Bob earns indifferent rewards if values of the above two rewards equals to each other; that is,

$$p \times R_b^{HH} + (1-p) \times R_b^{SH} = p \times R_b^{HS} + (1-p) \times R_b^{SS}. \quad (6)$$

After solving equation (6), we get the probability p as follows.

$$p = \frac{R_b^{SS} - R_b^{SH}}{R_b^{HH} + R_b^{SS} - R_b^{SH} - R_b^{HS}} \quad (7)$$

Using the same idea, we get the value of q , which is the probability that Bob chooses the honest mining strategy. The earned rewards earned by Alice R_a and Bob R_b are shown in equations (8) and (9) respectively.

$$R_a = \frac{R_a^{HH} R_a^{SS} - R_a^{HS} R_a^{SH}}{R_a^{HH} + R_a^{SS} - R_a^{SH} - R_a^{HS}} \quad (8)$$

$$R_b = \frac{R_b^{HH} R_b^{SS} - R_b^{HS} R_b^{SH}}{R_b^{HH} + R_b^{SS} - R_b^{SH} - R_b^{HS}} \quad (9)$$

D. The proposed game theory based rational mining strategy

A game theory based rational mining strategy in blockchains with two rational miners are proposed based on the three types of payoff matrices. We describe the proposed mining strategy in the point of view of Alice. Given the mining rates of Alice r_a and Bob r_b , the rational mining strategy is as follows.

- 1) If $r_a < 0.22$, Alice shall use honest mining strategy. If $r_a > 0.25$, Alice shall use selfish mining strategy.
- 2) If $0.22 \leq r_a \leq 0.25$ and $r_b < 0.22$, Alice shall use honest mining strategy since Bob will use honest mining strategy. If $0.22 \leq r_a \leq 0.25$ and $r_b > 0.25$, Alice shall use selfish mining strategy since Bob will use selfish mining strategy.

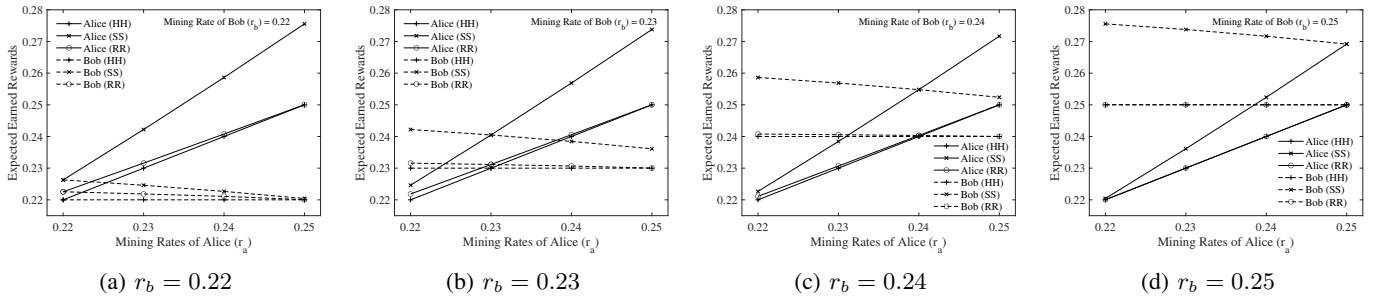


Fig. 1: Expected rewards earned by Alice and Bob under both honest, selfish, or rational mining strategies

- 3) If $0.22 \leq r_a, r_b \leq 0.25$, Alice shall use the mixed strategy with the probability distribution obtained from equations (7).

IV. DISCUSSIONS OF EARNED REWARDS

In this section, we discuss the rewards earned by Alice and Bob when they employed the proposed game theory based rational mining strategy. When at least one miner with mining rate outside the range of 0.22 to 0.25, the earned rewards of miners can be easily obtained since there is only one Nash equilibrium in the payoff matrix. However, when both the mining rates are between 0.22 and 0.25, the rewards earned by the game theory based rational mining strategy shall be calculated according a probability distribution as in equations (8) and (9).

Fig. 1 shows the rewards earned by Alice and Bob where $0.22 \leq r_a, r_b \leq 0.25$. In the figure, the earned rewards are labeled as *HH*, *SS* and *RR* if the miners both employ honest, selfish, and proposed game theory based rational mining strategies respectively. From the figure, we can make the following observations:

- The rewards earned by the game theory based rational mining strategy are between those earned by honest and selfish mining strategies. The rewards earned by both honest and selfish mining strategies provides the lower and upper bounds of those earned by the proposed mining strategy.
- Even though the most earned rewards occur when both miner use selfish mining strategy, rewards earned by the proposed mining strategy will approach to those earned by honest mining strategy when the mining rate approaches to 0.25.
- Given the fixed mining rate of Bob r_b , the rewards earned by Alice significantly increased with the increasing mining rates of Alice r_a . However, rewards earned by Bob slightly decreased with the increasing mining rates of Alice r_a .

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we proposed a game theory based rational mining strategy in a blockchain with two rational miners. Accurate analytical models are used to calculate the earned

rewards of two rational miners. By using the game theory based concepts, payoff matrices are constructed and the Nash equilibria are found. The game theory based rational mining strategy are developed according the the existence of Nash equilibria.

In most situations, the miners have dominant strategy and are able to choose the best strategy. When the mining rates of both miners are between 0.22 and 0.25, a mixed strategy is proposed since there is no dominant strategy for each miner. Numerical results show the rewards earned by the proposed rational mining strategy are always slightly larger than those earned by honest strategy.

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APPENDIX A

PAYOFF MATRICES UNDER DIFFERENT STRATEGY COMBINATIONS

We first show the payoff matrices where the mining rates are between 0.1 and 0.4 in Table V. In Table V, both miners have a dominant strategy. More details of the payoff matrices where the mining rates between 0.20 and 0.27 are shown in Table VI. In Table VI, the payoff matrices are classified into three types which are discussed in Section III.

TABLE V: Payoff Matrices when $0.1 \leq r_a, r_b \leq 0.4$ with Nash Equilibria

Payoff Matrices		Bob's Strategy								
		r_b	0.1		0.2		0.3		0.4	
r_a		Honest	Selfish	Honest	Selfish	Honest	Selfish	Honest	Selfish	
Alice's Strategy	0.1	Honest	0.100,0.100	0.103 ,0.072	0.100,0.200	0.102 ,0.182	0.100 ,0.300	0.096,0.327	0.100 ,0.400	0.079,0.526
		Selfish	0.072, 0.103	0.078,0.078	0.072, 0.206	0.079,0.199	0.072,0.309	0.068, 0.359	0.072,0.412	0.054, 0.550
	0.2	Honest	0.200,0.100	0.206 ,0.072	0.200,0.200	0.204 ,0.182	0.200 ,0.300	0.192,0.327	0.200 ,0.400	0.158,0.526
		Selfish	0.182, 0.102	0.199,0.076	0.182, 0.204	0.199,0.199	0.182,0.307	0.177, 0.369	0.182,0.409	0.136, 0.574
	0.3	Honest	0.300, 0.100	0.309,0.072	0.300, 0.200	0.307,0.182	0.300,0.300	0.289, 0.327	0.300,0.400	0.237, 0.526
		Selfish	0.327,0.096	0.359 ,0.068	0.327,0.192	0.369 ,0.177	0.327 ,0.289	0.341,0.341	0.327 ,0.385	0.262,0.555
	0.4	Honest	0.400, 0.100	0.412,0.072	0.400, 0.200	0.409,0.182	0.400,0.300	0.385, 0.327	0.400,0.400	0.316, 0.526
		Selfish	0.526,0.079	0.550 ,0.054	0.526,0.158	0.574 ,0.136	0.526 ,0.237	0.555,0.262	0.526 ,0.316	0.455,0.455

TABLE VI: Payoff Matrices when $0.20 \leq r_a, r_b \leq 0.27$ with Nash Equilibria

Payoff Matrices		Bob's Strategy								
		r_b	0.20		0.21		0.22		0.23	
r_a		Honest	Selfish	Honest	Selfish	Honest	Selfish	Honest	Selfish	
Alice's Strategy	0.20	Honest	0.200,0.200	0.204 ,0.182	0.200,0.210	0.204 ,0.195	0.200,0.220	0.203 ,0.209	0.200,0.230	0.202 ,0.222
		Selfish	0.182, 0.204	0.199,0.199	0.182, 0.215	0.198,0.214	0.182,0.225	0.198, 0.230	0.182,0.235	0.196, 0.246
	0.21	Honest	0.210,0.200	0.215 ,0.182	0.210,0.210	0.214 ,0.195	0.210,0.220	0.213 ,0.209	0.210,0.230	0.212 ,0.222
		Selfish	0.195, 0.204	0.214,0.198	0.195, 0.214	0.213,0.213	0.195,0.224	0.212, 0.229	0.195,0.234	0.211, 0.245
	0.22	Honest	0.220,0.200	0.225,0.182	0.220,0.210	0.224,0.195	0.220,0.220	0.223,0.209	0.220,0.230	0.222,0.222
		Selfish	0.209, 0.203	0.229 ,0.197	0.209, 0.213	0.229 ,0.212	0.208,0.223	0.228,0.228	0.208,0.233	0.227,0.245
	0.23	Honest	0.230,0.200	0.235,0.182	0.230,0.210	0.234,0.195	0.230,0.220	0.233,0.209	0.230,0.230	0.232,0.222
		Selfish	0.222, 0.202	0.246 ,0.196	0.222, 0.212	0.245 ,0.211	0.222,0.222	0.244,0.227	0.222,0.232	0.243,0.243
	0.24	Honest	0.240,0.200	0.245,0.182	0.240,0.210	0.244,0.195	0.240,0.220	0.244,0.209	0.240,0.230	0.243,0.222
		Selfish	0.236, 0.201	0.263 ,0.195	0.236, 0.211	0.262 ,0.210	0.236,0.221	0.262,0.225	0.236,0.231	0.261,0.242
	0.25	Honest	0.250,0.200	0.256,0.182	0.250,0.210	0.255,0.195	0.250,0.220	0.254,0.209	0.250,0.230	0.253,0.222
		Selfish	0.250,0.200	0.281 ,0.193	0.250,0.210	0.281 ,0.208	0.250 ,0.220	0.280,0.223	0.250 ,0.230	0.279,0.240
	0.26	Honest	0.260,0.200	0.266,0.182	0.260, 0.210	0.265,0.195	0.260,0.220	0.264,0.209	0.260,0.230	0.263,0.222
		Selfish	0.265,0.299	0.300 ,0.191	0.265,0.209	0.300 ,0.205	0.265 ,0.219	0.299,0.221	0.265 ,0.229	0.299,0.237
	0.27	Honest	0.270, 0.200	0.276,0.182	0.270, 0.210	0.275,0.195	0.270, 0.220	0.274,0.209	0.270, 0.230	0.273,0.222
		Selfish	0.280,0.197	0.319 ,0.189	0.280,0.207	0.320 ,0.203	0.280 ,0.217	0.320,0.218	0.280 ,0.227	0.320,0.234

Payoff Matrices		Bob's Strategy								
		r_b	0.24		0.25		0.26		0.27	
r_a		Honest	Selfish	Honest	Selfish	Honest	Selfish	Honest	Selfish	
Alice's Strategy	0.20	Honest	0.200,0.240	0.201 ,0.236	0.200,0.250	0.200,0.250	0.200 ,0.260	0.199,0.265	0.200 ,0.270	0.197,0.280
		Selfish	0.182,0.245	0.195, 0.263	0.182,0.256	0.193, 0.281	0.182,0.266	0.191, 0.300	0.182,0.279	0.188, 0.319
	0.21	Honest	0.210,0.240	0.211 ,0.236	0.210,0.250	0.210,0.250	0.210 ,0.260	0.209,0.265	0.210 ,0.270	0.207,0.280
		Selfish	0.195,0.244	0.210, 0.263	0.195,0.255	0.208, 0.281	0.195,0.265	0.206, 0.300	0.195,0.275	0.203, 0.320
	0.22	Honest	0.220,0.240	0.221,0.236	0.220,0.250	0.220, 0.250	0.220 ,0.260	0.219, 0.265	0.220 ,0.270	0.217, 0.280
		Selfish	0.209,0.244	0.225,0.262	0.209,0.254	0.224,0.280	0.209,0.264	0.221,0.299	0.209,0.274	0.218,0.320
	0.23	Honest	0.230,0.240	0.231,0.236	0.230,0.250	0.230, 0.250	0.230 ,0.260	0.229, 0.265	0.230 ,0.270	0.227, 0.280
		Selfish	0.222,0.243	0.242,0.261	0.222,0.253	0.240,0.279	0.222,0.263	0.238,0.299	0.222,0.273	0.234,0.320
	0.24	Honest	0.240,0.240	0.241,0.236	0.240,0.250	0.240, 0.250	0.240 ,0.260	0.238, 0.265	0.240 ,0.270	0.237, 0.280
		Selfish	0.236,0.241	0.259,0.250	0.236,0.251	0.257,0.278	0.236,0.261	0.255,0.297	0.236,0.272	0.252,0.319
	0.25	Honest	0.250,0.240	0.251,0.236	0.250,0.250	0.250, 0.250	0.250 ,0.260	0.248, 0.265	0.250 ,0.270	0.247, 0.280
		Selfish	0.250 ,0.240	0.278,0.257	0.250 ,0.250	0.276,0.276	0.250 ,0.260	0.274,0.295	0.250 ,0.270	0.270,0.318
	0.26	Honest	0.260, 0.240	0.261,0.236	0.260, 0.250	0.260, 0.250	0.260,0.260	0.258, 0.265	0.260,0.270	0.257, 0.280
		Selfish	0.265 ,0.238	0.298,0.255	0.265 ,0.248	0.296,0.274	0.265 ,0.258	0.293,0.293	0.265 ,0.268	0.290,0.315
	0.27	Honest	0.270, 0.240	0.272,0.236	0.270, 0.250	0.270, 0.250	0.270,0.260	0.268, 0.265	0.270,0.270	0.266, 0.280
		Selfish	0.280 ,0.237	0.319,0.252	0.280 ,0.247	0.317,0.270	0.280 ,0.257	0.315,0.290	0.280 ,0.266	0.312,0.312