On the Experimental Performance Measurement of Quantum Teleportation

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***Abstract*—Quantum information system has a potential to revolutionize the way we think of computing and communication. Quantum Teleportation has emerged as a promising technique for transmitting quantum information securely over long distances between quantum computers. However, it is unclear how well this performs in a real world. This paper attempts to evaluate the performance of Quantum Teleportation on actual quantum computers. The impact of environmental factors such as number of qubits, number of gates and hardware configuration on the Quantum Teleportation is measured over IBM Superconducting Quantum Computers. The paper aims to illuminate the current state of Quantum Teleportation and the associated challenges in measuring its performance, particularly in the context of security within quantum communication.**

***Index Terms*—Quantum Teleportation, Entanglement, Quantum Gates, Quantum Networking, Quantum Systems,**

I. Introduction

The field of quantum computing is becoming a cornerstone for solving many computational problems that are otherwise extremely difficult or impossible to tackle using classical computers [1]. Quantum computing utilizes quantum bits, also called qubits, which can exist in a superposition of states, unlike binary bits that can represent only one out of two states. Because qubits serve as the most basic information unit, quantum computers are exponentially faster than classical computers for many computation problems. Another interesting property of qubits is entanglement, a quantum phenomenon where the properties of particles become correlated to each other in such a way that their quantum states cannot be described independently, even if they are separated.

The principle of entanglement is crucial for quantum communication, enabling the secure transfer of information over long distances [2]. Among entangled particles, the measurement or observation of one set of particles determines the state of another set. Utilizing quantum entanglement allows the creation of secure channels. If an attacker measures one particle, the entanglement is disturbed, leading to an immediate detection of the phenomenon by the communicating parties. These properties have led to the development of secure information channels [2] over long distances using quantum teleportation [3]. Quantum teleportation is a technique where quantum states from one particle are transferred to another [4]. It opens up opportunities in various fields, including Quantum Sensors and Quantum Games [5]. Quantum network protocols [6] and devices [7] [8] [9]. Quantum teleportation plays a critical role in making the quantum internet a reality.

While various communication protocols, such as BB84 and B93 [10], have been introduced to enhance security by detecting eavesdroppers in communication channels, quantum teleportation has garnered significant interest among researchers worldwide. It is being explored for practical applications in information security and quantum computing. In quantum communication, particularly in the context of security applications, Quantum Teleportation is a favored method for securely transmitting quantum states across long distances. This protocol is especially valuable when maintaining the integrity and confidentiality of data is of utmost importance. Yet, current research lacks a valuable evaluation of the performance of quantum teleportation in the real world, which can shed much-needed light on the practical challenges [11] and the current state of networked communication using real quantum hardware. Existing work is limited in scope since it only covers single qubit state transfers and theoretical evaluation.

This paper presents an attempt to evaluate and demonstrate the performance of quantum teleportation on actual quantum hardware in more complex and realistic scenarios compared to existing work. The experiment is conducted on real IBM Quantum Systems, which are superconducting Quantum Computers. Experiments are carried out by varying the number of qubits and the number of gates on different IBM quantum hardware. The experiments are performed on multiple qubit state transfers using randomized states. Error rates in the form of the probability of observed states show the extent to which Quantum Teleportation is impacted by these parameters in a real networked quantum hardware environment. The paper addresses challenges in performance measurements of Quantum Teleportation over real quantum information systems.

The remainder of the paper is structured as follows: Section II provides an overview of the background & related work. Section III details the experimental setup and methodologies. Section IV discusses the experimental findings. The paper is concluded in Section V with future research directions.

II. Background and Related Work

This section briefly describe Quantum Teleportation, and existing notable experimentation work on this topic. Furthermore, we distinguish our work from the existing ones.

A. Quantum Teleportation

Quantum Teleportation enables the end-to-end secure transfer of single-qubit data. Its efficiency is attributed to its

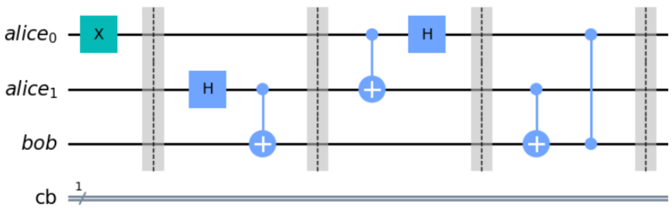

Fig. 1: Quantum Teleportation: single qubit state transfer

utilization of the unique properties associated with the four Bell states as explained in next few paragraphs.

Teleportation is illustrated in Figure 1. The state $|\Psi_a\rangle$ represents the quantum state to be transferred, and Alice and Bob share a pair of entangled qubits using the state $|\phi^-\rangle$. The transfer process is represented by the following equations:

$$|\Psi_a\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

$$|\phi^-\rangle = (|00\rangle + |11\rangle)/\sqrt{2} \tag{2}$$

$$|\Psi_0\rangle = |\Psi_a\rangle|\phi^-\rangle \tag{3}$$

After the Bell State Measurement, the resulting state is:

$$\begin{aligned} |\Psi_1\rangle = [&|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\ &+ |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)] \end{aligned} \tag{4}$$

Where $|\Psi_a\rangle$, $|\phi^-\rangle$, and $|\Psi_0\rangle$ play central roles: $|\Psi_a\rangle$ represents the quantum state Alice wants to teleport, described by α and β. $|\phi^-\rangle$ is the shared entangled pair between Alice and Bob, a Bell state with $|00\rangle$ and $|11\rangle$. Item $|\Psi_0\rangle$ denotes the quantum state of the entire circuit, combining $|\Psi_a\rangle$ and $|\phi^-\rangle$.

After preparing these states, Alice performs a Bell state measurement on her qubits, collapsing them into one of four Bell states ($|00\rangle$, $|01\rangle$, $|10\rangle$, or $|11\rangle$) with equal probabilities. She sends the measurement outcomes (two classical bits) to Bob. Bob, situated remotely, receives these classical bits. Depending on the message received, he applies specific quantum gates to his qubit: For $|00\rangle$, Bob does nothing. For $|01\rangle$, he applies an X-gate. For $|10\rangle$, he applies a Z-gate. For $|11\rangle$, he applies both X and Z gates. These gates transform Bob's qubit into the desired state $|\Psi_a\rangle$ initially held by Alice by manipulating the state of Bob's qubit to match the original quantum state $|\Psi_a\rangle$ that Alice intended to teleport.

In this manner, quantum teleportation enables the secure transfer of quantum states from one location to another, harnessing shared entangled pairs and employing classical communication to guarantee the reliable reconstruction of the desired quantum state at the receiving end and playing a pivotal role in ensuring the security and integrity of quantum communication and quantum computing systems.

B. *Experiments with Quantum Teleportation*

In 1997, experiments on Quantum Teleportation began [12], four years after the protocol's conception by Anton Zelinger et.al. in Vienna. They successfully teleported an entangled state of light, transferring a qubit from one node to another using a pair of photons created through Parametric Down Conversion (PDC). To perform Bell State Measurement, a device was used to recognize the $|\psi^-\rangle$ state between Alice's initial message and her qubit with entanglement. Another significant experiment conducted at Caltech the following year [13] employed Quantum Continuous Variable (QCV) encoded in different states of light and demonstrated teleportation for all Bell States. Teleportation over a 143 km optical link was achieved using an active feed-forward method [14]. The experiment employed diverse techniques, including frequency-uncorrelated polarization entangled photon pair sources, collinear photon pair sources, clock synchronization for entanglement, and single-photon detectors. Additionally, ground-to-satellite communication was demonstrated [3], utilizing multi-photon entanglement and narrow beam divergence. Successful teleportation of single-photon qubits occurred over a distance of 1400 km to a low Earth orbit satellite.

Existing work involves the demonstration of Quantum Teleportation where as our work focuses on performance measurement of the same. In this work, real quantum computers are used to transfer qubits states using Teleportation. It is to be noted that there are different types of teleportation such as Quantum Teleportation of Continuous Variables, Port-based Teleportation, and Quantum Gate Teleportation. In our experiment, we used used Gate teleportation [5]. Ours is the first work that aims to measure the performance of Quantum Teleportation for multi qubit state transfers in a realistic scenarios on real quantum computers.

III. Experiment Methodologies and Setup

The primary focus of the study centers on the analysis of error rates and the likelihood of successful data transfers as key performance metrics. This section outlines the experimental methodologies employed to assess the performance of Quantum Teleportation. It provides insights into the experimental parameters and setups, including quantum system configurations such as the quantity of gates, the number of qubits involved, and the diverse hardware configurations characterized by T1 and T2 times. The prevalent networked scenarios, which include the transfer of single-qubit quantum states, the transmission of randomized multi-qubit states, and the conveyance of entangled multi-qubit states are also described.

A. *Quantum System Hardware*

There are various implementations of quantum computers, including Superconducting, Diamond NV centers, Photonic, and Trapped ions. This paper utilizes IBM Quantum Systems, superconducting Quantum Computers[1], which are briefly described below. These quantum systems were chosen because of their adherence to the DiVincenzo Criteria for scalable quantum systems, encompassing well-characterized qubits, the ability to initialize quantum states, sufficient decoherence time

[1] See https://quantum-computing.ibm.com/ for a complete description

TABLE I: Configuration of IBM Quantum Computers used in the experiment.

System	Avg T1 time	Avg T2 time	Frequency	Gate Time	Prob meas0 prep1	Prob meas1 prep0	CNOT Error
ibmq_belem	93.44us	107.13us	5.246GHz	465.778ns	0.0458	0.01	1.42E-02
ibmq_oslo	124.53us	104.76us	5.047GHz	337.778ns	0.0166	0.0146	1.20E-02

for quantum stability, a universal set of quantum gates for circuit decomposition, and qubit measurement capabilities.

The decoherence time (referred to as T2 time) is the duration during which qubits lose their quantum properties. Conversely, the relaxation time (referred to as T1 time) represents the time taken by a qubit to dissipate energy into its surroundings, transitioning from state $|1\rangle$ to state $|0\rangle$. Longer relaxation and decoherence times correlate with lower error rates. To ensure satisfactory T1 and T2 times, qubits are isolated from their environment, making state transitions challenging.

In this paper, two distinct IBM quantum computers, Belem and Oslo, are used (Table I shows the machine configuration). Various parameters are considered, as previously elucidated in relation to T1 and T2 times. Each qubit is connected to a microwave resonator with distinct frequencies, as indicated in the table. Gate time serves as another parameter, signifying the duration of multi-qubit gate operations. It is important to clarify that this parameter does not encompass single-qubit gates. Prob meas0 prep1 signifies the probability of transitioning to state 0 after state 1 is prepared, while Prob meas1 prep0 denotes the probability of transitioning to state 1 after state 0 is prepared. The CNOT error reflects the errors introduced by the two-qubit gates employed within the circuit.

B. Parameters and Set up

Following parameters are taken into account for their known influence on the quantum information transfer error rate:

- Number of Gates: The quantity of gates employed varies by augmenting the number of X-gates (see Figure 1).
- Number of Qubits: This parameter represents the fundamental unit of data transfer.
- Diverse Hardware Configurations: Distinct hardware setups, characterized by T1 and T2 times, are explored as they significantly impact qubit decay times.

The evaluation encompasses three networked scenarios tailored to assess Quantum Teleportation's performance:

- Single Qubit: The circuit depicted in Figure 1 demonstrates the transfer of a single-qubit quantum state from Alice to Bob, with a simplified representation involving a bit flip gate to showcase error rates.
- Randomized Multi-Qubit: Figure 2 presents the circuit for a scenario involving two qubits.
- Entangled Multi-Qubit: The circuit in Figure 3 facilitates the transfer of entangled two-qubit states.

These experiments employ fictional characters, Alice and Bob, as communicating entities to illustrate the state transfer process. In each of the circuit scenarios, error rates and transfer probabilities serve as critical performance metrics for assessing the effectiveness of Quantum Teleportation

IV. Experimental Results and Discussions

This section presents and discusses the experimental results and challenges of the measurement of quantum teleportation.

A. Teleporting one qubit quantum state

Figure 1 shows the circuit designed for the transfer of a single-qubit quantum state from Alice to Bob. Initially, Alice sends a qubit in the state $|1\rangle$ to Bob. Alice and Bob share an entangled pair of qubits, with the first qubit containing the quantum message to be transmitted. Bell State Measurement is applied to Alice's qubits to effect the transfer to Bob. The circuit is executed on the IBM Quantum Computer named belem with 1024 measurement shots. The potential outcomes of the circuit are $|100\rangle$, $|101\rangle$, $|110\rangle$, and $|111\rangle$. The qubit ordering is from right to left, meaning that $|100\rangle$ signifies that Bob's qubit is in state $|1\rangle$ while Alice's second and first qubits are in state $|0\rangle$. All possible outcomes share the common feature that Bob's qubit is in state $|1\rangle$, demonstrating the successful teleportation of the quantum state. Alice's qubit undergoes a change as per the No-Cloning theorem [1], the quantum states cannot be copied from one location to another.

To assess the influence of the number of gates on the quantum teleportation process, we systematically applied the X-gate, repeating the procedure multiple times. The circuit was executed ten times, with 1024 measurement shots for each run. The resulting error probabilities corresponding to different outcomes for Bob's qubit are illustrated in Figure 4. Analyzing the data presented in Figure 4, a noteworthy trend emerges. There is a significant reduction in error rates when employing nine X-gates compared to instances with only one or three X-gates. Despite this, the overall trend indicates that error rates exhibit minor fluctuations as the number of gates in the circuit is increased or decreased. This observation leads us to infer that the relationship between the number of gates and error rates is not a simple linear one, contrary to our expectations.

B. Teleporting multi qubit quantum state

The quantum circuit for this particular configuration, as illustrated in Figure 2, is designed to transfer two-qubit quantum states from Alice to Bob. In this scenario, Alice utilizes four qubits: two for encoding the message and two for establishing entanglement with Bob. An entangled pair is created between Alice and Bob, and two random quantum states from Alice are destined for Bob's reception. To facilitate the state transfer, we employ Bell State Measurement on Alice's qubits. Notably, due to the absence of classically conditioned operations support in IBM Quantum Systems, we leverage the principle of deferred measurement, allowing us to move measurements from intermediate stages to the end of the

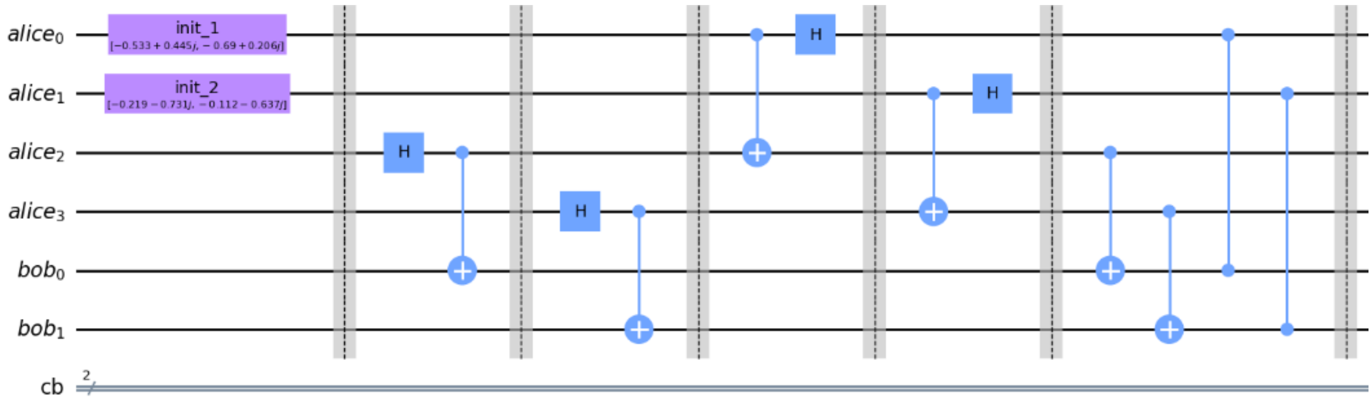

Fig. 2: Quantum Teleportation Circuit for randomized multi qubit state transfer

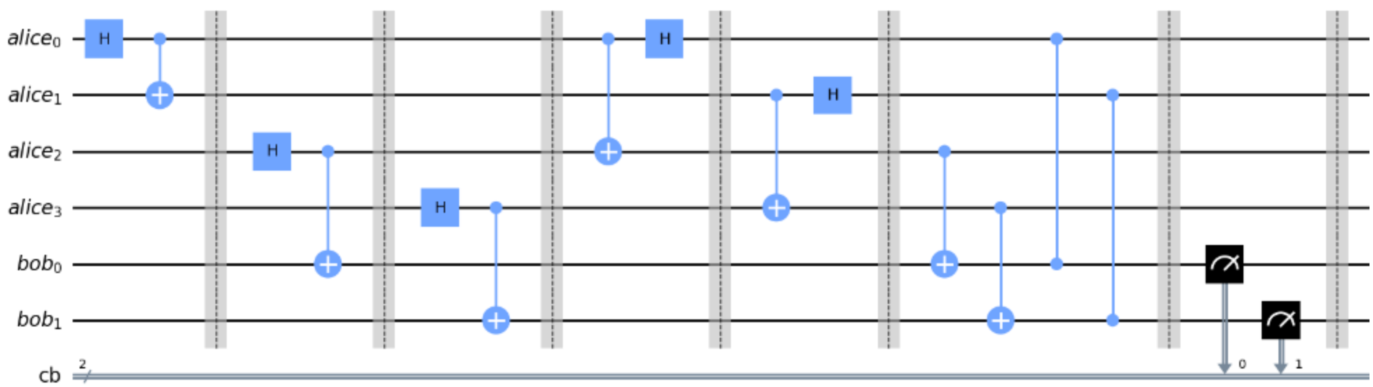

Fig. 3: Quantum Teleportation circuit for transfer of entangled multi qubit state transfer

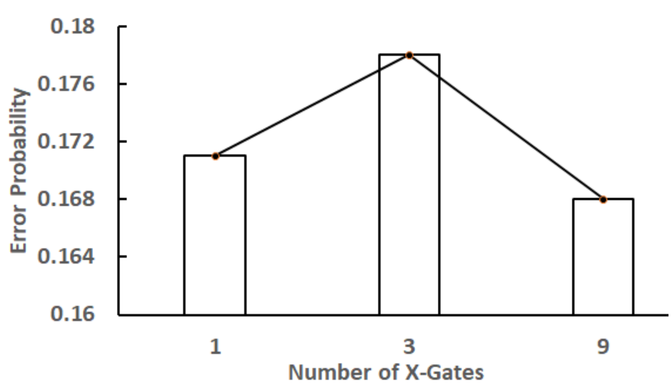

Fig. 4: Impact of number of X-gates on Error rate for single qubit state transfer

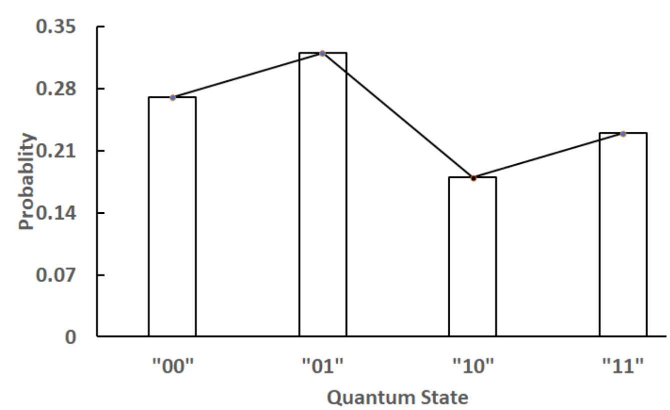

Fig. 5: Transfer probability of two random quantum states

circuit. Any classically conditioned operations are substituted with quantum conditioned operations to achieve consistent outcomes. To validate the results, we executed the circuit on the ibmq oslo system, a seven-qubit platform, for 4000 shots. Figure 5 presents the probability distributions for Bob's qubits. The qubit ordering is reversed in this context.

Our experimental observations highlight an intriguing aspect of quantum teleportation. Contrary to the expectation of equal probabilities for all combinations of qubit states, our results reveal deviations, quantified as errors in the quantum teleportation process. These errors manifest as deviations from the theoretically ideal probability of 0.25 for each possible outcome. This observation underscores the sensitivity of quantum teleportation to the number of qubits involved in the process. However, it is crucial to acknowledge that the observed deviation can be attributed to various contributing factors all related to the inherent challenges associated with achieving perfect entanglement.

C. *Teleporting entangled state*

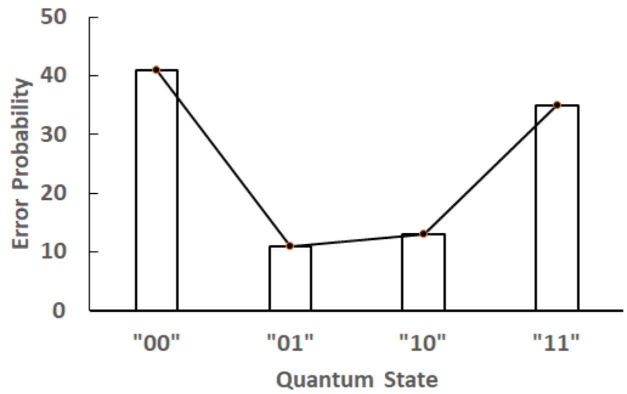

Fig. 6: Teleported entangled-pair state

The circuit depicted in Figure 3 illustrates the quantum teleportation of an entangled state. In this particular scenario, a set of entangled states, specifically the EPR state $|\psi^-\rangle$, is transmitted from Alice to Bob. To assess the performance of this entangled state transfer, the circuit is executed on the IBM Quantum System in Oslo, generating data from 4000 measurement shots. Figure 6 provides insight into the probability distribution of the transmitted entangled state pairs.

An essential factor to scrutinize is the probability of correctly measuring Bob's state in the $|\psi^-\rangle$ state, depicted in Figure 6. Within this context, the probabilities of 01 and 10 assume significance, as they signify errors or deviations from the anticipated successful transfer of entangled states to Bob. The emergence of probabilities 01 and 10 highlights potential deviations from the intended outcome and underscores the complexity of achieving accurate transfers in quantum communication and computation systems. This analysis underscores the pressing need for ongoing advancements in quantum error correction and noise mitigation strategies to enhance the fidelity of such transfers in real-world quantum applications.

V. Conclusion

Quantum Computing is growing rapidly, providing solutions to the problems that are hard to solve with a classical computers. Quantum Networking is a promising field and Quantum Teleportation holds a key to make it a reality. However, to build networking protocols using Quantum Teleportation, we must be able to understand the performance and current state of Quantum Teleportation over real quantum computers. In this paper, we made an attempt to understand the impact of various factors such as number of gates, qubits and hardware on the performance of Quantum Teleportation.

Our experiments provide insights into the performance of quantum teleportation circuits on real IBM quantum systems. Our observations reveal a nuanced relationship between error rates and the number of gates, challenging the notion of a straightforward correlation. Additionally, we have observed deviations from the anticipated uniformity in the reception of all possible combinations of qubit states, along with errors in achieving the expected successful transfer of entangled states. Several factors may contribute to this behavior, including gate imperfections and the intricate interplay of quantum states. Further research is needed to precisely quantify these effects and optimize gate configurations for improved teleportation performance. Therefore, the performance measurement of the quantum teleportation remains a challenging issue and requires more attention from the research community. Without it, it is hard to design new protocols which can perform in the wild to make quantum communication and networking a reality.

Our future work is on improvement of the performance and scalability of Quantum Teleportation experiments on the real quantum systems exploring the root causes for deviations presented in this paper. Extending the size and complexity of the quantum experiments with an aim to discover practical challenges in designing, developing and, deployment of the communication protocols using Quantum Teleportation on real quantum information systems is a worthy long term goal.

Acknowledgment

We thank IBM and their Quantum team for access and support during our experiments on IBM Quantum systems.

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