Fast Frequency-Direction Mapping Design for Data Communication With True-Time-Delay Array Architecture

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***Abstract*—In recent times, beamforming architectures utilizing True-Time-Delay arrays have gained significant interest in the field of wide-band data communication due to their abilities to generate frequency-dependent responses with great flexibility. This paper proposes a new heuristic fast algorithm with the goal of beamforming signals to different directions in different sub-bands. The proposed algorithm not only achieves better spectral efficiency than existing heuristic algorithms, but also significantly reduces computation time compared to optimization-based methods.**

***Index Terms*—Millimeter-wave, true time delay, analog beamforming**

I. Introduction

Millimeter wave (mmW) and terahertz (THz) communication have become a promising technology for future wireless network due to large amount of spectrum resources [1]. In order to address the significant propagation loss in the strength of mmW signals, transmit and receive beamforming technique is proposed. This method produces spatially focused transmission, referred as beams, with high array gain in a specific direction [2]. Due to the large number of antennas in mmW and THz system, fully digital beamforming technique is becoming prohibitively expensive since each antenna needs to be connected to a dedicated Radio Frequency (RF) chain. In order to reduce hardware cost and energy consumption, hybrid beamforming technique is proposed, which uses analog components such as phase-shifters and switches to reduce the number of RF chains [3].

However, there is a noteworthy limitation in these systems: analog components used in hybrid beamforming can only generate frequency-independent responses, which severely reduce the flexibility of designing beam patterns especially in wide-band systems. For example, current mmW phased antenna array and hybrid architectures can not divide the entire bandwidth into smaller portions and allocate them to different users located in distinct directions. This is a crucial use case because it has the potential to increase system capacity, achieve low latency, and improve spectral efficiency.

As a potential solution to the aforementioned limitations, the true-time-delay (TTD) array architecture has gained considerable attention as a notable array design technology, particularly in wide-band systems. A number of studies have investigated the use of TTDs in wide-band systems to mitigate the beam squint effects and generate directional pencil beams, thereby enhancing the quality for data communication [4], [5]. The use of TTDs for one-shot beam training has been explored in [6]–[8]. Furthermore, a TTD-based rainbow beam multiple access method has been proposed to greatly expand the angular coverage by dispersing different sub-carriers in various directions [9]–[11].

Recently, a new architecture called Joint Phase Time Array (JPTA) [12] is proposed to achieve desired frequency-direction mapping with digital beamforming based on iterative optimization. However, it requires a large amount of computation time, which is impractical in real-time systems. Also, another method called mmFlexible [13] is developed to create frequency-dependent beams. The authors derived the analytical solution to the design of delays and phases based on a least squares problem, which significantly reduces the computation time. However, mmFlexible suffers from severe performance reduction in wide-band systems because the beam squint is not considered.

In this paper, we propose a fast heuristic algorithm based on modulo operators to design delays and phases, and analyze the performance of existing methods. In summary, the contributions of this paper are as follows:

- We propose a fast heuristic algorithm in TTD array architecture to design delays and phases to achieve desired frequency-direction mapping based on modulo operators.
- We analyze the limitation of the proposed algorithm in serving multiple (more than two) users in different directions simultaneously.
- We evaluate the performance of existing algorithms for creating frequency-direction beamforming responses, in terms of spectral efficiency and computation time.

The rest of paper is organized as follows. In Section II, we present the TTD based architecture and the statement of the problem. Section III-A includes an optimization based algorithm to design desired delays and phases, followed by a heuristic algorithm based on modulo operators in Section III-B. The numerical results including the spectral efficiency and computation time are shown in Section IV. Finally, Section V concludes the paper.

Notation: Scalars, vectors, matrices and sets are represented by non-bold lowercase, bold lowercase, bold uppercase, light-case calligraphic letters respectively. Additionally, $\mathrm{j} = \sqrt{-1}$.

The transpose and Hermitian transpose of $\mathbf{A}$ are $\mathbf{A}^T$ and $\mathbf{A}^H$ respectively. $|a|$ represents the absolute value of scalar a, $\|\mathbf{a}\|$ represents the L-2 norm of vector $\mathbf{a}$. $\angle\mathbf{a}$ refers to the vector of element-wise phase angles of a vector $\mathbf{a}$.

II. System Model and Problem Statement

This section introduces the system model of the proposed architecture and the problem statement.

We consider a wireless communication system in which a single Base-Station (BS) serves multiple users (UEs), operating with bandwidth BW around a center frequency f_c. The BS is equipped with a uniform linear antenna array having N_T elements and a single RF chain. The antenna spacing is set at half-wavelength at the center frequency f_c, and each of the N_T antennas is equipped with a dedicated phase-shifter and a TTD element as shown in Fig. 1. The phase-shifts and time delays are designed to serve two UEs located at angles θ_1 and θ_2.

It is assumed that the TTDs can be adjusted continuously within their delay variation range. The phase-shifters have unit magnitude and can have their phases adjusted to any value between $-\pi$ and π. The transmission is performed using Orthogonal Frequency Division Multiplexing (OFDM) waveform with M subcarriers indexed as $\mathcal{M} = \{1, 2, \ldots, M\}$.

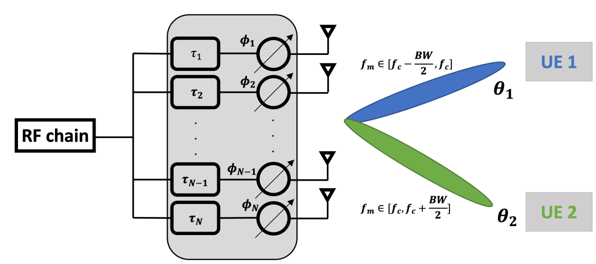

Fig. 1. Illustration of the proposed system architecture.

The $N_T \times 1$ downlink transmit signal on sub-carrier $m \in \mathcal{M}$ $\mathbf{x}_m$can be formulated as follows:

$$\mathbf{x}_m = \underbrace{\frac{1}{\sqrt{N_T}} \begin{bmatrix} e^{j\phi_1} & 0 & \cdots & 0 \\ 0 & e^{j\phi_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\phi_{N_T}} \end{bmatrix}}_{\mathbf{\Phi}} \underbrace{\begin{bmatrix} e^{-j2\pi f_m \tau_1} \\ e^{-j2\pi f_m \tau_2} \\ \vdots \\ e^{-j2\pi f_m \tau_{N_T}} \end{bmatrix}}_{\mathbf{w}_{TTD}[m]} s_m$$
$$= \mathbf{\Phi}\mathbf{w}_{TTD}[m] s_m \tag{1}$$

where s_m is the scalar data on the m-th sub-carrier, τ_n is the time delay of n-th TTD element, ϕ_n is the phase of the phase-shifter connected to the n-th antenna. The $N_T \times N_T$ diagonal matrix $\mathbf{\Phi}$ represents the effect of phase-shifters and the $N_T \times 1$ frequency-dependent vector $\mathbf{w}_{TTD}[m]$ captures the effects of TTD elements for the m-th sub-carrier. f_m is the frequency of the m-th sub-carrier, which is defined as follows:

$$f_m = \begin{cases} f_c - \frac{BW}{2} + \frac{m}{M} BW & 0 \le m < \frac{M}{2} \\ f_c + \frac{(m - \frac{M}{2})}{M} BW & \frac{M}{2} \le m < M \end{cases} \tag{2}$$

To evenly split the available frequency range into two sub-bands and allocate each sub-band to individual users in various directions, we specify intended beam patterns based on the sub-carrier index. The desired target beam behaviors $\mathbf{b}_m$ are shown as follows:

$$\mathbf{b}_m = \begin{cases} \mathbf{a}_m(\theta_1) & 0 \le m < \frac{M}{2} \\ \mathbf{a}_m(\theta_2) & \frac{M}{2} \le m < M \end{cases} \tag{3}$$

where the lower half of the signal band is assigned to serve the user in angle θ_1, the upper half band is assigned to serve the user in angle θ_2. The impact of beam squint cannot be ignored in this system, as it involves a large number of antennas and a wide bandwidth. Thus, the array response vector $\mathbf{a}_m(\theta)$ at the BS for a given angle θ can be expressed as:

$$\mathbf{a}_m(\theta) = \left[\begin{matrix} 1 & e^{j\frac{\pi \sin(\theta) f_m}{f_c}} & \cdots e^{j\frac{(N_T - 1)\pi \sin(\theta) f_m}{f_c}} \end{matrix}\right]^{\mathrm{T}} \tag{4}$$

In this work, we focus on how to set TTD values and phase shifts to achieve high performance with low computational complexity. The expression of normalized beamforming gain at direction θ for m-th sub-carrier can be achieved through simple mathematical manipulation. It can be expressed as follows:

$$G(\theta, m) = |\mathbf{a}_m^H(\theta)\mathbf{\Phi}\mathbf{w}_{TTD}[m]|^2$$
$$= \frac{1}{N_T} \left| \sum_{n=1}^{N_T} e^{-j2\pi f_m \tau_n + j\phi_n - j\frac{(n-1)\pi \sin(\theta) f_m}{f_c}} \right|^2 \tag{5}$$

To assess the effectiveness of various algorithms, it is essential to have an appropriate criterion for evaluation. In this study, we adopt spectral efficiency as the metric to evaluate the performance, which can be computed in the following manner:

$$R = \frac{1}{M} \sum_{m=1}^{M} \log_2 \left(1 + \frac{\rho}{\sigma^2} |\mathbf{b}_m^H \mathbf{\Phi}\mathbf{w}_{TTD[m]}|^2\right) \tag{6}$$

where ρ is the transmit power for each sub-carrier, σ^2 is the power of the additive white Gaussian noise, $\frac{\rho}{\sigma^2}$ is expressed as signal-to-noise ratio (SNR).

III. Algorithm Design

In this section, we provide two methods to design delays and phases. The first algorithm is based on least squares, aiming at minimizing the phase distance between the analog beamformer and the desired phase pattern, and can be extend to multiple users. The second algorithm is based on modulo operator, focusing on offering a favorable trade-off between the communication performance and computation time.

A. Least Squares

In this subsection, we provide an analytical solution to design time delays and phase shifts based on least squares.

For 2-beam case, we can formulate the optimization problem as follows:

$$\underset{\tau, \phi}{\operatorname{argmax}} \quad \sum_{m=1}^{M} G(\theta, m) \tag{7}$$

where the parameter θ meets the following requirements:

$$\begin{cases}\theta = \theta_1, 1 \leq m \leq \frac{M}{2} \\ \theta = \theta_2, \frac{M}{2} < m \leq M\end{cases} \tag{8}$$

Inspired by [5] and [12], the optimization above can be reformulated as follows:

$$\underset{\tau_n,\phi_n}{\operatorname{argmin}} \quad \sum_{m=1}^{M}(-2\pi f_m\tau_n + \phi_n - \mathscr{U}(\angle[\mathbf{b}_m]_n))^2 \tag{9}$$

as an approximation of the original problem. It transforms the problem from maximizing the beam pattern gain to minimizing the phase distance between the target beam and the analog beamformer. The function denoted as $\mathscr{U}$ is recognized as a phase unwrapping function, which restricts the phase difference between adjacent sub-carriers.

This formulation is a least squares problem and can be solved analytically. By taking the gradient and setting the gradient to zero, the solution to the problem can be derived as follows:

$$\tau_n = \frac{ad_n - b_n}{1 - ac}, \quad \phi_n = \frac{d_n - b_n c}{1 - ac} \tag{10}$$

where

$$\begin{cases} a = \frac{\sum_{m=1}^{M} f_m}{2\pi\sum_{m=1}^{M} f_m^2} \\ b_n = \frac{\sum_{m=1}^{M} f_m \mathscr{U}(\angle[\mathbf{b}_m]_n)}{2\pi\sum_{m=1}^{M} f_m^2} \\ c = \frac{2\pi}{M}\sum_{m=1}^{M} f_m \\ d_n = \frac{1}{M}\mathscr{U}(\angle[\mathbf{b}_m]_n) \end{cases} \tag{11}$$

This method can extend two-beam case to multiple-beam case by changing the desired beam behaviors $\mathbf{b}_m$. However, in multiple-beam case, if the directions of the users are given arbitrarily, there is a decrease in performance across certain sub-bands due to the limitation of the analog components.

B. Modulo based Heuristic Algorithm

Based on the optimization results reported in [12] as well as the least squares solution, we make an observation that the value of TTDs can be represented as a superposition of modulo time delays and uniform linear delays. Additionally, [13] provided mathematical proof that the phase shifts and time delays strictly follow the modulo form for two symmetric directions (without considering beam squint). These findings offer valuable insights into potential methods for generating time delays and phase shift value, which may reduce computation time without the need for optimization tools.

We assume that TTD value is a superposition of modulo delays and uniform linear delays, which can be expressed as follows:

$$\tau_n = \alpha(n-1) \mod \frac{\beta}{BW} + \gamma(n-1) \tag{12}$$

where α, β and γ are parameters that control the beam patterns. The roles of parameter α and β are shown in Fig. 2, by setting $\phi_n = 0$. We can judge by intuition that α determines the angular separation between adjacent beams, while β determines the bandwidth of each sub-band. The proposed heuristic modulo algorithm can achieve the desired frequency-direction mapping through three steps.

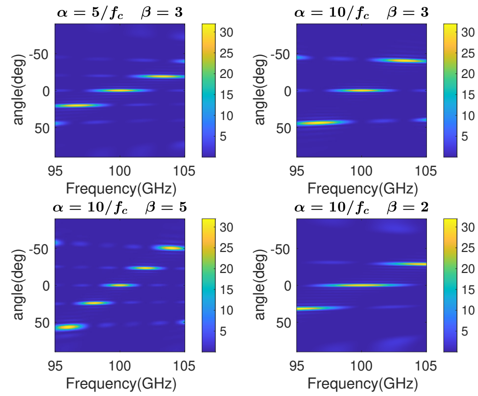

Fig. 2. Illustration of the impacts of parameter α and β

Step1 is to find the sector rainbow beam, passing through $(f_{\rm L}, \theta_1)$ and $(f_{\rm U}, \theta_2)$, where $f_{\rm L}$ and $f_{\rm U}$ are defined as follows:

$$f_{\rm L} = [\lfloor(\frac{f_c}{BW} - \frac{1}{2})\beta\rfloor + 1]\frac{BW}{\beta} \tag{13}$$

$$f_{\rm U} = f_{\rm L} + \frac{BW}{\beta} \tag{14}$$

where β is a pre-determined parameter. The method for obtaining suitable β value will be discussed and elaborated in the following.

To obtain the sector rainbow beam, the following equations need to be solved:

$$\begin{cases} -2\pi f_{\rm L}\tau_n^{(1)} + \phi_n^{(1)} = \pi\frac{f_{\rm L}}{f_c}\sin\theta_1(n-1) \\ -2\pi f_{\rm U}\tau_n^{(1)} + \phi_n^{(1)} = \pi\frac{f_{\rm U}}{f_c}\sin\theta_2(n-1) \end{cases} \tag{15}$$

The solution to Equation (15) is as follows:

$$\tau_n^{(1)} = (f_{\rm L}\sin\theta_1 - f_{\rm U}\sin\theta_2)\frac{\beta}{2BWf_c}(n-1) \tag{16}$$

$$\phi_n^{(1)} = \pi(f_{\rm L} + f_{\rm U})\tau_n^{(1)} + (f_{\rm L}\sin\theta_1 + f_{\rm U}\sin\theta_2)\frac{\pi}{2f_c}(n-1) \tag{17}$$

Step2 introduces modulo operators to ensure that sub-carriers within a sub-band are pointing to the same direction. It is noteworthy to mention that for specific f_m, if $\frac{f_m\beta}{BW} \in \mathbb{Z}$, modulo delays cause the same array response as uniform linear TTD array.

$$\tau_n^{(2)} = \tau_n^{(1)} \mod \frac{\beta}{BW} \tag{18}$$

$$\phi_n^{(2)} = \phi_n^{(1)} \tag{19}$$

Step3 gives us a method to shift the beam pattern on frequency-direction mapping. In phase domain, the following equations hold:

$$\begin{cases} -2\pi f_{\rm L}\tau_n^{(2)} + \phi_n^{(2)} = \pi\frac{f_{\rm L}}{f_c}\sin\theta_1(n-1) + 2k_1\pi, k_1 \in \mathbb{Z} \\ -2\pi f_{\rm U}\tau_n^{(2)} + \phi_n^{(2)} = \pi\frac{f_{\rm U}}{f_c}\sin\theta_2(n-1) + 2k_2\pi, k_2 \in \mathbb{Z} \end{cases} \tag{20}$$

In order to achieve the even distribution of bandwidth, we need to find a solution for the following equations:

$$\begin{cases} -2\pi(f_\mathrm{L}-\Delta f)\tau_n^{(3)}+\phi_n^{(3)}=\pi\frac{f_\mathrm{L}-\Delta f}{f_c}\sin\theta_1(n-1)+2k_1\pi \\ -2\pi(f_\mathrm{U}-\Delta f)\tau_n^{(3)}+\phi_n^{(3)}=\pi\frac{f_\mathrm{U}-\Delta f}{f_c}\sin\theta_2(n-1)+2k_2\pi \end{cases} \tag{21}$$

where $\Delta f=\frac{1}{2}(f_\mathrm{L}+f_\mathrm{U})-f_c$ serves as a measure of the degree to which the beam pattern shifts on frequency-direction mapping.

The solution to Equation (21) is as follows:

$$\tau_n^{(3)}=\tau_n^{(2)}+\frac{\Delta f}{2f_c(f_\mathrm{L}-f_\mathrm{U})}(\sin\theta_1-\sin\theta_2)(n-1) \tag{22}$$

$$\phi_n^{(3)}=2\pi(f_\mathrm{L}-\Delta f)\tau_n^{(3)} - 2\pi f_\mathrm{L}\tau_n^{(2)}+\phi_n^{(2)}-\pi\frac{\Delta f}{f_c}\sin\theta_1(n-1) \tag{23}$$

In conclusion, TTD parameters that control the beam pattern can be summarized as follows:

$$\begin{cases} \alpha=(f_\mathrm{L}\sin\theta_1-f_\mathrm{U}\sin\theta_2)\frac{\beta}{2BWf_c} \\ \gamma=\frac{\Delta f}{2f_c(f_\mathrm{L}-f_\mathrm{U})}(\sin\theta_1-\sin\theta_2) \end{cases} \tag{24}$$

We take $\theta_1=-\pi/4$, $\theta_2=\pi/6$ as an example to illustrate the procedure of generating desired beam patterns, which is shown in Fig. 3.

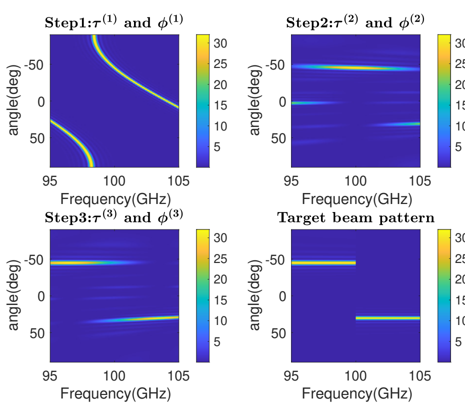

Fig. 3. Illustration of the procedures of the proposed algorithm to create a desired beam pattern with $\theta_1=-\pi/4$ and $\theta_2=\pi/6$

For a two-beam case, the ideal value of β depends on several factors, such as the number of antennas, sub-carriers, desired angle pairs, and the ratio of bandwidth to center frequency. It is impractical to re-evaluate and choose a new β value every time the angle changes because of the extensive computational resources required for this task. To address this issue, we propose a statistical method to select an appropriate β value which is optimal in a statistical sense.

Due to the beam squint, shifting the beam pattern in frequency-direction mapping usually causes severe performance loss. One effective way to select β value wisely is to avoid beam pattern shift, which means that $\Delta f=0$. Based on this assumption, we can have a sequence of discrete β value, which satisfies the equation below:

$$\beta=(n+\frac{1}{2})\frac{BW}{f_c} \tag{25}$$

where $\frac{f_c}{BW}-\frac{1}{2}\leq n\leq\frac{2f_c}{BW}-\frac{1}{2}, n\in\mathbb{Z}$. For given specific θ_1 and θ_2, the spectral efficiency versus different β value is shown in Fig. 4, where $N_T=64$ and $\frac{BW}{f_c}=0.05$.

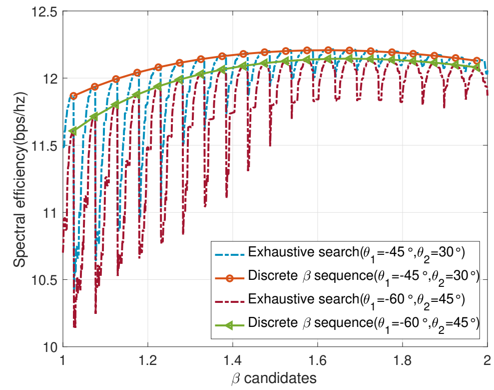

Fig. 4. Spectral efficiency versus different β value.

It is easy to select the optimal discrete β value based on Equation (25) as a approximation of optimal β value. We can perform Monte Carlo trials and select suitable β value based on the distribution of optimal discrete β. Table. I is a look-up table for different system settings.

TABLE I
β SELECTION LOOK-UP TABLE

N_T	M	BW/f_c	β
32	1024	0.1	1.55
32	2048	0.1	1.55
64	1024	0.1	1.75
64	1024	0.05	1.525

Multiple-user(more than 2) case can also be derived using modulo operators by increasing the value of β. Without losing generality, we can assume that $\tau_n=\alpha(n-1) \mod \frac{\beta}{BW}$, $\phi_n=0$. For every f_m that satisfies $\frac{f_m\beta}{BW}\in\mathbb{Z}$, we can easily prove that the corresponding direction for sub-carrier f_m satisfies the equation below:

$$f_m\alpha+\frac{f_m}{2f_c}\sin\theta_m\in\mathbb{Z} \tag{26}$$

where θ_m represents the direction that sub-carrier f_m is pointing to. It indicates that due to the constraints of analog components, the directions of the users are constrained.

IV. Numerical Results

In this section, we simulate a downlink system, which operates at a carrier frequency of 100GHz and utilizes 1024 sub-carriers, with a system bandwidth set to 10GHz. The BS is equipped with $N_T=64$ uniform linear antennas, unless specified otherwise. We compare proposed algorithms with existing algorithms including JPTA iterative optimization, JPTA heuristic algorithm [12] and mmFlexible [13]. The numerical results are simulated on AMD Ryzen Threadripper 3970X 32-Core Processor using MATLAB 2022b.

A. Spectral Efficiency

Fig. 5 presents the distribution of spectral efficiency over the entire testing angle pairs for $N_T = 64$ and $\frac{BW}{f_c} = 0.1$. θ_1 and θ_2 are uniformly distributed in $[-\frac{\pi}{3}, \frac{\pi}{3}]$. It is observed that the proposed algorithm is an acceptable approximation of the achievable rate generated by the JPTA iterative algorithm.

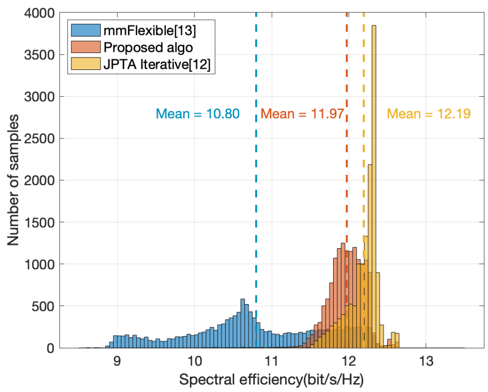

Fig. 5. Distribution of different algorithms for $N_T = 64, BW/f_c = 0.1$, M=1024 at SNR=20dB

Fig. 6 presents the average spectral efficiency under different numbers of antennas, when $\frac{BW}{f_c} = 0.1$ and SNR=20dB. We can find that JPTA iterative optimization can achieve 12.193 bps/Hz on average whereas the least squares solution comes very close to the JPTA iterative algorithm. Proposed heuristic modulo algorithm with selected β value according to the look-up table can attain 11.973 bps/Hz, while if the β value is set to 1.5, the spectral efficiency will drop to 11.678 bps/Hz. The mmFlexible algorithm suffers from severe beam squint because the algorithm is designed without considering beam squint. When the number of antenna is 64, mmFlexible can achieve 10.800 bps/Hz. The JPTA heuristic can achieve 11.335 bps/Hz when the correction for beam squint is added to the algorithm. However, when the number of antennas is relatively low, other algorithms tend to perform slightly better compared to the modulo-based algorithm.

We analyze spectral efficiency of the proposed heuristic modulo algorithm versus the ratio of bandwidth to center frequency from 0.005 to 0.1, in order to analyze how these algorithms perform in a wide band system. We can observe from Fig. 7 that JPTA iterative optimization algorithm and least squares solution exhibits comparable performance when the ratio increases. Although the proposed heuristic modulo algorithm exhibits a 1.5% decrease in performance when comparing to JPTA iterative algorithm, it still provides a significant performance improvement over other heuristic algorithms. The

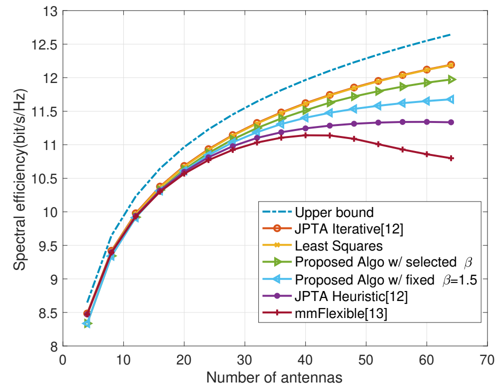

Fig. 6. Spectral efficiency versus N_T for $BW/f_c = 0.1$ at SNR=20dB

nonmonotonicity of the proposed algorithm with fixed $\beta = 1.5$ is caused by the beam pattern shift and the beam squint, same reason in Fig. 4. It is also important to emphasize that the performance of the system is related to ratio of the bandwidth to carrier frequency, which indicates that the algorithm is applicable for all frequency bands.

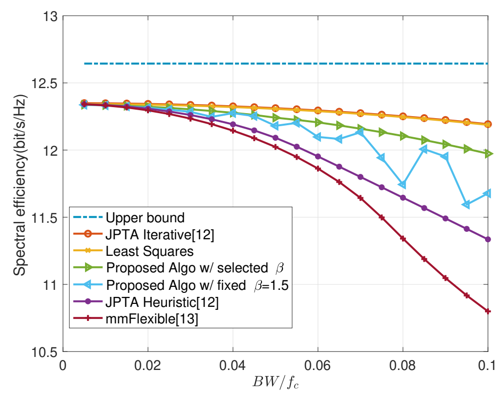

Fig. 7. Spectral efficiency versus BW/f_c for $N_T = 64$ at SNR=20dB

B. Computational Complexity

We herein analyze the computational complexity of the proposed algorithm and other algorithms mentioned above. The complexity for JPTA is $\mathcal{O}(NKM)$ while least square has $\mathcal{O}(KM)$, where N represents the the number of iterations, K represents the number of sub-carriers and M refers to the number of antennas. Both the proposed algorithm and mmFlexible exhibit a computational complexity of $\mathcal{O}(M)$, which is significantly less than that of optimization-based approaches.

In Fig. 8 and Fig. 9, we compare the execution time of the considered scheme, which is important in real-time system. According to Fig. 8, it is noticeable that JPTA iterative algorithm and least squares algorithm are M-dependent. As the number of sub-carriers increases, the computation time increases linearly. Other heuristic algorithms are M-independent and if we select the β value according to Equation (25), the final step of the proposed heuristic modulo algorithm, which is beam pattern shift can be skipped and it further reduces the computation time. Of all the algorithms considered, this particular algorithm exhibits the minimal computation time, thereby demonstrating superior computational efficiency in comparison to its counterparts.

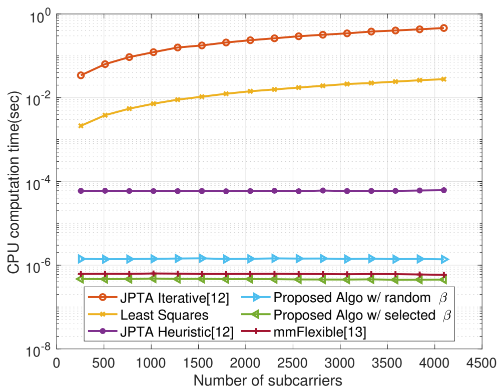

Fig. 8. CPU runtime versus M for $N_T = 32$

Finally, we show the CPU runtime versus the number of antennas.The computational complexity of all the algorithms mentioned increases linearly with respect to the number of antennas N_T. The max iteration number for JPTA is set to 10. We can find that for a scheme with 64 antennas and 1024 sub-carriers, the proposed algorithm takes 5×10^{-7}s to compute the time delays and phase shifts, while mmFlexible takes 8×10^{-7}s and JPTA iterative algorithm takes up to 0.24s.

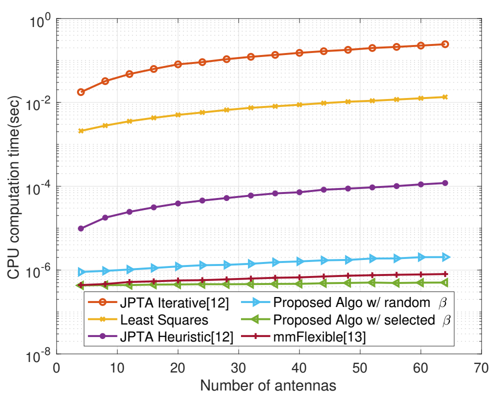

Fig. 9. CPU runtime versus N_T for M=1024

Based on the simulation results, it has been found that both the JPTA iterative algorithm and the least squares based algorithm exhibit the highest achievable rate compared to other approaches. Nevertheless, it is worth noting that the computational complexity of these two algorithms depend on the number of sub-carriers, which results in relatively longer computation time. On the other hand, the proposed heuristic modulo algorithm exhibits a robust ability to resist beam squint compared to other heuristic algorithms. Furthermore, it offers a favorable trade-off between spectral efficiency and computation time, which can be significant in real-time systems.

V. Conclusion

In this work, we propose a new algorithm with low computation complexity to realize frequency-dependent analog beamforming. We also analyze the existing algorithms for the scheme that communicate with 2 users in arbitrary directions. It is shown that the proposed algorithm can dramatically reduce computation overhead at the cost of a small amount of performance loss compared to existing optimization-based algorithms. While it is possible to achieve a scenario involving multiple users, the specific angles that can be attained are limited in systems catering to more than two users. This limitation is imposed by the constraints of analog components.

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