Beamforming Design for MIMO DFRC Systems with Transmit Covariance Constraints

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***Abstract*—This paper develops the optimal beamforming design in a downlink multi-user multi-input-multi-output (MIMO) dual-function radar-communication (DFRC) system, which maximizes the weighted sum-rate of communicating users under the prescribed transmit covariance constraint for radar performance guarantee. Specifically, we exploit the connection between weighted sum-rate and weighted minimum-mean-squared-error (MMSE) to reformulate the problem, and develop a block-coordinate-descent (BCD) type algorithm to iteratively compute the transmit beamforming and receive beamforming solutions. Using this approach, we reveal that the optimal receive beamforming is the classic MMSE one and the optimal transmit beamforming design solves an orthogonal Procrustes problem (OPP), thereby allowing for closed-form solutions to subproblems in each BCD step and fast convergence of the proposed algorithm to a high-quality overall beamforming design. Numerical results validate the effectiveness of our proposed scheme.**

***Index Terms*—Dual-function radar-communication, beamforming design, block coordinate descent.**

I. Introduction

To meet the explosive demand for high-speed mobile data services, radar spectrum bands such as S-band (2–4 GHz) and C-band (4–8 GHz) are nowadays shared with communication systems. For this reason, the concept of integrated sensing and communications (ISAC) has drawn growing attention [1], [2]. To enable communication and radar spectrum sharing in the same band, one approach is joint design for co-existence of radar and communication systems, where radar systems and communication systems operating on different devices exchange side-information for cooperation, leading to extra hardware and energy cost and raised system complexity [3]–[5]. Toward a more efficient system design, another approach is development of dual-function radar-communication (DFRC) systems, where the same set of signals is used for sensing and communications on the same hardware platform [6].

Some existing literature on DFRC systems has generally aimed to optimize radar performance under communication performance constraints, leading to a communication-centric design. In [6], [7], semidefinite relaxation (SDR) based approaches were developed to minimize the radar beampattern matching errors subject to communicating users' quality-of-service constraints. Furthermore, [8] proposed to optimize Cramér-Rao bound (CRB) for improvement of the target estimation performance under communication performance constraints, and [9] developed joint transceiver beamforming designs to maximize the signal to interference-plus-noise ratio (SINR) at radar receivers. However, the above-mentioned designs [6]–[9] could only offer best-effort radar sensing performance. The sensing performance loss was inevitable.

By contrast, radar-centric DFRC system designs attempted to embed communication information into radar signals to achieve dual functionality without degrading radar performance. In [10], information sequences were embedded into radar pulses by employing waveform diversity and sidelobe control, but spatial multiplexing was not exploited. The sensing performance of multi-input-multi-output (MIMO) radars highly relies on the transmit covariance of waveforms [11]. A few recent works [12], [13] optimized the communication performance under a prescribed transmit covariance constraint preserving the radar performance. Optimal transmit waveform/beamforming designs were put forth under the instantaneous covariance constraint in [12] or under the average covariance constraint in [13] for multi-input-single-output (MISO) systems. However, these existing methods [12], [13] cannot be readily extended to more general multi-user multi-stream MIMO systems with the additional need for receive beamforming design.

In this paper, we consider a downlink MIMO DFRC system where a base station (BS) simultaneously probes multiple targets and performs multi-user communications. For this MIMO DFRC system, we develop the optimal beamforming design that maximizes the weighted sum-rate for communicating users under the prescribed transmit covariance constraint to guarantee radar performance.

The contributions of the paper are summarized as follows.

- We exploit the connection between weighted sum-rate and weighted minimum mean squared error (MMSE) to reformulate the intended problem and develop an efficient block-coordinate-descent (BCD)-type method to iteratively refine the transmit and receive beamformers.
- With the proposed BCD approach, we reveal that the optimal receive beamforming takes the classic MMSE principle and the optimal transmit beamforming amounts to solving an orthogonal Procrustes problem (OPP). Closed-form solutions can then be derived for the transmit

Work in this paper was supported by the National Natural Science Foundation of China under Grants No. 62231010 and No. 62071126, and the Innovation Program of Shanghai Municipal Science and Technology Commission Grant No. 21XD1400300.

and receive beamformers in each BCD step, ensuring fast convergence of our algorithm to a high-quality overall beamforming design.

- Simulations demonstrate that the proposed BCD algorithm converges rapidly to an efficient beamforming solution that performs at least 40% better than benchmark schemes under the medium-to-high signal-to-noise ratio (SNR) settings.

The rest of this paper is organized as follows. Section II describes the system model. Section III develops an efficient BCD-type algorithm to obtain a high-performance beamforming design. Simulation results are provided in Section IV, followed by the conclusions in Section V.

Notation: Bold-face lower-case letters are used for vectors and bold-face upper-case letters represent matrices; for a matrix, $(\cdot)^H$, $\mathrm{Tr}(\cdot)$, $\det(\cdot)$ and $||\cdot||_F$ denote its conjugate transpose, trace, determinant, and Frobenius norm, respectively; $\mathbf{I}_n$ denotes the $n \times n$ identity matrix; $\mathbb{C}^{m\times n}$ denotes the set of $m \times n$ complex matrices; $\mathrm{Re}\{\cdot\}$ takes real part; $\mathbb{E}\{\cdot\}$ denotes ensemble expectation.

II. System Model

We consider a MIMO DFRC system where a BS simultaneously probes J far-field targets and communicates with K downlink users at the same time. Suppose that the BS is equipped with N_{tx} transmit antennas. Let $\mathbf{F}_k \in \mathbb{C}^{N_{\mathrm{tx}}\times d}$ denote the beamforming matrix for (multi-stream) communication symbol vector $\mathbf{c}_k(t) \in \mathbb{C}^d$ at time t from BS to user k. Without loss of generality, we assume that $\mathbb{E}\{\mathbf{c}_k(t)\mathbf{c}_k{}^H(t)\} = \mathbf{I}_d$ and the communication symbols of different users are generated independently. Overall, the communication symbol vector $\mathbf{c}(t) = [\mathbf{c}_1^T(t), \ldots, \mathbf{c}_K^T(t)]^T$ containing $D = d \times K$ data streams is precoded linearly using $\mathbf{F}_{\mathrm{c}} = [\mathbf{F}_1, \mathbf{F}_2, ..., \mathbf{F}_K] \in \mathbb{C}^{N_{\mathrm{tx}}\times D}$.

For sensing purpose, a radar signal vector $\mathbf{r}(t) \in \mathbb{C}^{N_{\mathrm{tx}}-D}$ consisting of $(N_{\mathrm{tx}} - D)$ independently and pseudo-randomly generated symbols [7], is transmitted along with $\mathbf{c}(t)$. Assume that $\mathbb{E}\{\mathbf{r}(t)\mathbf{r}^H(t)\} = \mathbf{I}_{N_{\mathrm{tx}}-D}$, and the radar signals are uncorrelated with the communication symbols, i.e., $\mathbb{E}\{\mathbf{r}(t)\mathbf{c}^H(t)\} = \mathbf{0}_{(N_{\mathrm{tx}}-D)\times D}$. The radar symbols are precoded using a beamforming matrix $\mathbf{F}_{\mathrm{r}} \in \mathbb{C}^{N_{\mathrm{tx}}\times(N_{\mathrm{tx}}-D)}$.

With the $N_{\mathrm{tx}} \times N_{\mathrm{tx}}$ beamforming matrix $\mathbf{F} = [\mathbf{F}_{\mathrm{c}}, \mathbf{F}_{\mathrm{r}}]$, the transmit signal vector $\mathbf{x}(t) \in \mathbb{C}^{N_{\mathrm{tx}}}$ is given by

$$\mathbf{x}(t) = \mathbf{F}\begin{bmatrix} \mathbf{c}(t) \\ \mathbf{r}(t) \end{bmatrix} = \mathbf{F}_{\mathrm{c}}\mathbf{c}(t) + \mathbf{F}_{\mathrm{r}}\mathbf{r}(t),\ t = 0, 1, \cdots. \quad (1)$$

A. Radar Performance Guarantee

For MIMO radar, the sensing performance highly relies on the transmit beampattern. Based on the signal model (1), the transmit beampattern is determined by the covariance of transmit signals:

$$\mathbf{R}_x = \mathbb{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{F}\mathbf{F}^H = \mathbf{F}_{\mathrm{c}}\mathbf{F}_{\mathrm{c}}^H + \mathbf{F}_{\mathrm{r}}\mathbf{F}_{\mathrm{r}}^H. \quad (2)$$

Achieving the desired beampattern then amounts to a transmit covariance constraint for the beamforming matrix. Based on the specific applications and performance metrics of interest, a desired transmit covariance matrix $\mathbf{R}_{\mathrm{des}}$ can be determined in advance [11]. To ensure an acceptable sensing performance, we require the transmit covariance to match the prescribed $\mathbf{R}_{\mathrm{des}}$; i.e.,

$$\mathbf{F}\mathbf{F}^H = \mathbf{R}_{\mathrm{des}}. \quad (3)$$

It is worth noting that the transmit covariance constraint (3) also accounts implicitly for a power constraint for the transmit beamforming matrix $\mathbf{F}$. Given $\mathbf{F}$, the total transmit power is clearly given by $P = \mathrm{Tr}(\mathbf{F}\mathbf{F}^H) = \mathrm{Tr}(\mathbf{R}_{\mathrm{des}})$.

B. Communication Performance Metric

Suppose that each user is equipped with N_{rx} ($d \leqslant N_{\mathrm{rx}} \leqslant N_{\mathrm{tx}}$) receive antennas. For downlink communications in the DFRC system, the received signal of user k is the mixture of its own signal, the interference from the other users, the radar signal, and the noise; i.e.,

$$\mathbf{y}_k(t) = \mathbf{H}_k\mathbf{F}_k\mathbf{c}_k(t) + \mathbf{H}_k\sum_{i\neq k}\mathbf{F}_i\mathbf{c}_i(t) + \mathbf{H}_k\mathbf{F}_{\mathrm{r}}\mathbf{r}(t) + \mathbf{n}_k(t), \quad (4)$$

where $\mathbf{H}_k \in \mathbb{C}^{N_{\mathrm{rx}}\times N_{\mathrm{tx}}}$ represents the channel matrix from the BS to user k, and $\mathbf{n}_k(t) \in \mathbb{C}^{N_{\mathrm{rx}}}$ denotes the additive white Gaussian noise (AWGN) with zero mean and covariance matrix $\sigma^2\mathbf{I}_{N_{\mathrm{rx}}}$. The noise is independent of communication and radar signals. With linear receive beamforming matrix $\mathbf{G}_k \in \mathbb{C}^{N_{\mathrm{rx}}\times d}$, the estimated signal can be expressed as

$$\hat{\mathbf{c}}_k(t) = \mathbf{G}_k^H\mathbf{y}_k(t). \quad (5)$$

We use a weighted sum of user rates as communication performance metric. Based on the well-known MIMO capacity formula, the achievable weighted sum-rate is given by

$$C = \sum_{k=1}^{K}\omega_k C_k, \quad (6)$$

where ω_k denotes the priority weight of user k and the achievable rate C_k of user k is given by

$$\begin{aligned} C_k = \log\det(\mathbf{I}_d + \mathbf{F}_k^H\mathbf{H}_k^H(\sigma^2\mathbf{I}_{N_{\mathrm{rx}}} + \sum_{i\neq k}\mathbf{H}_k\mathbf{F}_i\mathbf{F}_i^H\mathbf{H}_k^H \\ + \mathbf{H}_k\mathbf{F}_{\mathrm{r}}\mathbf{F}_{\mathrm{r}}^H\mathbf{H}_k^H)^{-1}\mathbf{H}_k\mathbf{F}_k). \end{aligned} \quad (7)$$

Note that to achieve the maximum rate in (7), the optimal MMSE beamforming matrix $\mathbf{G}_k^{\mathrm{mmse}}$ should be adopted in the receiver. Hence, the expression for the maximum achievable user rate here does not explicitly include the receive beamformer; it can be written merely as a function of $\mathbf{F}$.

Our goal is then to optimize $\mathbf{F}_{\mathrm{c}}$ and $\mathbf{F}_{\mathrm{r}}$, hence maximizing the weighted sum-rate (6) under the transmit covariance constraint (3).

III. Proposed Beamforming Design

We consider the beamforming design in a multi-user MIMO DFRC system. Based on (6) and (7), the intended optimization problem can be expressed as

$$\max_{\mathbf{F}} \sum_{k=1}^{K} \omega_k \log\det(\mathbf{I}_d + \mathbf{F}_k^H \mathbf{H}_k^H (\sigma^2 \mathbf{I}_{N_{\mathrm{rx}}} + \sum_{i\neq k} \mathbf{H}_k \mathbf{F}_i \mathbf{F}_i^H \mathbf{H}_k^H + \mathbf{H}_k \mathbf{F}_{\mathrm{r}} \mathbf{F}_{\mathrm{r}}^H \mathbf{H}_k^H)^{-1} \mathbf{H}_k \mathbf{F}_k) \quad (8a)$$

$$\text{s.t. } \mathbf{F}\mathbf{F}^H = \mathbf{R}_{\mathrm{des}}. \quad (8b)$$

Recall that the total transmit power constraint for $\mathbf{F}$ is incorporated into the transmit covariance constraint (8b). Hence, there is no need for an explicit transmit power constraint.

The problem (8) is clearly non-convex. Yet, notice that constraint (8b) indeed confines the beamforming design in a complex Grassmann manifold. Hence, one can develop a Riemannian gradient descent-based method to approximately solve (8) using the existing *Manopt* toolbox; see the derivation in Appendix A of our extended version [14].

Different from such a standard manifold optimization approach, we opt to a judicious reformulation to propose a more efficient BCD algorithm to solve (8) with better performance.

A. Problem Reformulation

To make the challenging non-convex problem (8) more tractable, we exploit the connection revealed in [15] between weighted sum-rate and weighted MMSE to reformulate (8) as follows.

Let $\mathbf{G} = [\mathbf{G}_1, \ldots, \mathbf{G}_K]$ collect the receive beamforming matrices. We define the mean squared error (MSE) matrix of user k as

$$\begin{aligned}
&\mathbf{E}_k(\mathbf{G}, \mathbf{F}) = \mathbb{E}\{(\hat{\mathbf{c}}_k(t) - \mathbf{c}_k(t))(\hat{\mathbf{c}}_k(t) - \mathbf{c}_k(t))^H\} \\
&= \mathbf{I}_d - 2\,\mathrm{Re}\{\mathbf{G}_k^H \mathbf{H}_k \mathbf{F}_k\} + \sum_{i=1}^{K} \mathbf{G}_k^H \mathbf{H}_k \mathbf{F}_i \mathbf{F}_i^H \mathbf{H}_k^H \mathbf{G}_k \\
&\quad + \mathbf{G}_k^H \mathbf{H}_k \mathbf{F}_{\mathrm{r}} \mathbf{F}_{\mathrm{r}}^H \mathbf{H}_k^H \mathbf{G}_k + \sigma^2 \mathbf{G}_k^H \mathbf{G}_k \\
&= \mathbf{I}_d - 2\,\mathrm{Re}\{\mathbf{G}_k^H \mathbf{H}_k \mathbf{F}_k\} + \mathbf{G}_k^H \mathbf{H}_k \mathbf{R}_{\mathrm{des}} \mathbf{H}_k^H \mathbf{G}_k + \sigma^2 \mathbf{G}_k^H \mathbf{G}_k,
\end{aligned} \quad (9)$$

where the last equality holds due to constraint (8b). Then we can transform the original problem (8) into a matrix-weighted sum-MSE minimization problem:

$$\min_{\mathbf{F},\mathbf{G},\mathbf{W}} \sum_{k=1}^{K} \omega_k (\mathrm{Tr}(\mathbf{W}_k \mathbf{E}_k) - \log\det(\mathbf{W}_k)) \quad (10a)$$

$$\text{s.t. } \mathbf{F}\mathbf{F}^H = \mathbf{R}_{\mathrm{des}}, \quad (10b)$$

where the weight matrices $\mathbf{W}_k \succeq \mathbf{0}$, $\forall k$, are auxiliary optimization variables. The equivalence between problems (8) and (10) can be established since the optimal solution $\mathbf{F}^*$ of the two problems are identical; more generally, $\mathbf{F}^*$ is a stationary point solution for (8) if and only if it is part of a stationary point solution for (10) [15], [16].

B. BCD Algorithm

Note that the optimization variables in problem (10) consist of $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{W}$. A BCD-based approach can be used to optimize $\mathbf{G}$, $\mathbf{W}$, and $\mathbf{F}$ alternately. Specifically, given fixed $\mathbf{F}$, the optimal receive beamformer is clearly provided by following the MMSE criterion:

$$\mathbf{G}_k^{\mathbf{mmse}} = (\mathbf{H}_k \mathbf{R}_{\mathrm{des}} \mathbf{H}_k^H + \sigma^2 \mathbf{I}_{N_{\mathrm{rx}}})^{-1} \mathbf{H}_k \mathbf{F}_k, \ \forall k. \quad (11)$$

With $\mathbf{F}$ and $\mathbf{G}$ fixed, we have an unconstrained convex problem concerning $\mathbf{W}$, for which the optimal solution is

$$\mathbf{W}_k^* = \mathbf{E}_k^{-1}, \ \forall k. \quad (12)$$

With $\mathbf{G}$ and $\mathbf{W}$ fixed, problem (10) becomes

$$\max_{\mathbf{F}} \sum_{k=1}^{K} \omega_k \mathrm{Re}\{\mathrm{Tr}(\mathbf{W}_k \mathbf{G}_k^H \mathbf{H}_k \mathbf{F}_k)\} \quad (13a)$$

$$\text{s.t. } \mathbf{F}\mathbf{F}^H = \mathbf{R}_{\mathrm{des}}. \quad (13b)$$

Here, (13b) is a non-convex quadratic equality constraint. Hence, the problem (13) is non-convex. Nevertheless, we next show that the globally optimal $\mathbf{F}^*$ can be actually obtained in a closed form for problem (13). Specifically, we first perform Cholesky decomposition on $\mathbf{R}_{\mathrm{des}}$ to obtain

$$\mathbf{R}_{\mathrm{des}} = \mathbf{L}\mathbf{L}^H, \quad (14)$$

where $\mathbf{L} \in \mathbb{C}^{N_{\mathrm{tx}} \times N_{\mathrm{tx}}}$ is a lower triangular matrix.

By substituting (14) into (13b), we can rewrite (13b) as

$$\mathbf{L}^{-1}\mathbf{F}\mathbf{F}^H\mathbf{L}^{-H} = \mathbf{I}_{N_{\mathrm{tx}}}. \quad (15)$$

Define $\tilde{\mathbf{F}} = \mathbf{L}^{-1}\mathbf{F}$ and $\tilde{\mathbf{H}}_k = \mathbf{H}_k\mathbf{L}$. We can then equivalently rewrite problem (13) as

$$\max_{\tilde{\mathbf{F}}} \sum_{k=1}^{K} \omega_k \mathrm{Re}\{\mathrm{Tr}(\mathbf{W}_k \mathbf{G}_k^H \tilde{\mathbf{H}}_k \tilde{\mathbf{F}}_k)\} \quad (16a)$$

$$\text{s.t. } \tilde{\mathbf{F}}\tilde{\mathbf{F}}^H = \mathbf{I}_{N_{\mathrm{tx}}}. \quad (16b)$$

With some algebraic manipulations, we can subsequently transform problem (13) into the following equivalent form

$$\max_{\tilde{\mathbf{F}}_{\mathrm{c}}} \mathrm{Re}\{\mathrm{Tr}(\mathbf{M}^H \tilde{\mathbf{F}}_{\mathrm{c}})\} \quad (17a)$$

$$\text{s.t. } \tilde{\mathbf{F}}_{\mathrm{c}}^H \tilde{\mathbf{F}}_{\mathrm{c}} = \mathbf{I}_D, \quad (17b)$$

where $\mathbf{M} = \left[\omega_1 \tilde{\mathbf{H}}_1^H \mathbf{G}_1 \mathbf{W}_1^H, \cdots, \omega_k \tilde{\mathbf{H}}_K^H \mathbf{G}_K \mathbf{W}_K^H\right]$. Due to (17b), problem (17) is indeed also equivalent to the following OPP:

$$\min_{\tilde{\mathbf{F}}_{\mathrm{c}}} ||\mathbf{M} - \tilde{\mathbf{F}}_{\mathrm{c}}||_F^2 \quad (18a)$$

$$\text{s.t. } \tilde{\mathbf{F}}_{\mathrm{c}}^H \tilde{\mathbf{F}}_{\mathrm{c}} = \mathbf{I}_D. \quad (18b)$$

It has been established in [17, Proposition 7] that problem (18) has a unique globally optimal solution in closed form. More explicitly, by performing singular value decomposition (SVD), i.e., $\mathbf{M} = \mathbf{U}_{\mathrm{M}} \mathbf{\Sigma}_{\mathrm{M}} \mathbf{V}_{\mathrm{M}}^H$, with the eigenvalues arranged in descending order along the diagonal of $\mathbf{\Sigma}_{\mathrm{M}}$, the unique globally optimal solution for problem (18) is given by

$$\tilde{\mathbf{F}}_{\mathrm{c}}^* = \mathbf{U}_{\mathrm{M}}(1:D)\mathbf{V}_{\mathrm{M}}^H, \quad (19)$$

where $\mathbf{U}_{\mathrm{M}}(1:D)$ collects the first D column vectors of $\mathbf{U}_{\mathrm{M}}$. According to (16b), the optimal radar beamformers $\tilde{\mathbf{F}}_{\mathrm{r}}^{*}$ should be in the null space of $\tilde{\mathbf{F}}_{\mathrm{c}}^{*}$. It then follows that

$$\tilde{\mathbf{F}}_{\mathrm{r}}^{*} = \mathbf{U}_{\mathrm{M}}(D+1:N_{\mathrm{tx}}), \tag{20}$$

where $\mathbf{U}_{\mathrm{M}}(D+1:N_{\mathrm{tx}})$ collects the last $(N_{\mathrm{tx}}-D)$ column vectors of $\mathbf{U}_{\mathrm{M}}$. With $\tilde{\mathbf{F}}^{*} = [\tilde{\mathbf{F}}_{\mathrm{c}}^{*}, \tilde{\mathbf{F}}_{\mathrm{r}}^{*}]$, we can in turn obtain the optimal $\mathbf{F}^{*} = \mathbf{L}\tilde{\mathbf{F}}^{*}$.

Algorithm 1 summarizes the overall procedure to solve (10).

Algorithm 1 BCD Algorithm for Solving (10) .

```
Initialize F = L.
Repeat
    Obtain G_k by (11), ∀k.
    Obtain W_k by (12), ∀k.
    Obtain F̃ using (19) and (20) , and update F = LF̃.
Until convergence.
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The convergence of Algorithm 1 is guaranteed since each BCD iteration monotonically decreases (the lower-bounded) objective function of problem (10) over a compact feasible set. Moreover, problem (10) features a differentiable objective function and a separable feasible set, i.e., the overall feasible set $\mathcal{F}(\mathbf{F},\mathbf{G},\mathbf{W}) = \mathcal{F}(\mathbf{F}) \times \mathcal{F}(\mathbf{G}) \times \mathcal{F}(\mathbf{W})$. It then follows from [18] that the BCD-based Algorithm 1 surely converges to at least a stationary point solution $(\mathbf{F}^*, \mathbf{G}^*, \mathbf{W}^*)$ of problem (10). As discussed earlier in Section III-A, the transmit beamforming matrix $\mathbf{F}^*$ in the stationary point solution $(\mathbf{F}^*, \mathbf{G}^*, \mathbf{W}^*)$ obtained for (10) is also a stationary point solution of (8).

Note that at each BCD step, we attain a unique closed-form solution for $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{W}$. Hence, the BCD-based Algorithm 1 has very low computational complexity. Numerical results next also show that Algorithm 1 can converge within only a few iterations and yield a beamforming solution with much better performance than that based on the manifold optimization (i.e., Riemannian gradient descent) approach.

IV. Numerical Results

We run Monte Carlo simulations to gauge the performance of the proposed schemes. Suppose that the BS and the users are equipped with uniform linear arrays with half-wavelength spacing between adjacent antennas. Assume a Rayleigh fading model, where each element of channel matrix $\mathbf{H}_k$ is independently generated according to the standard complex Gaussian distribution $\mathcal{CN}(0,1)$. The total transmit power is P and the transmit SNR is defined as P/σ^2.

Assume that the radar system directs beams towards $J=3$ targets of interest located at $\theta_1 = -60^\circ$, $\theta_2 = 0^\circ$, and $\theta_3 = 60^\circ$. The desired transmit covariance $\mathbf{R}_{\mathrm{des}}$ is determined by solving a constrained least-squares problem to minimize the radar beampattern matching errors as in [11], where the ideal beampattern consisting of three $\Delta = 9^\circ$ main beams, is given by

$$\tilde{P}_d(\theta) = \begin{cases} 1, & \text{if} \quad \theta_j - \Delta/2 \leqslant \theta \leqslant \theta_j + \Delta/2, j=1,2,3, \\ 0, & \text{otherwise.} \end{cases}$$

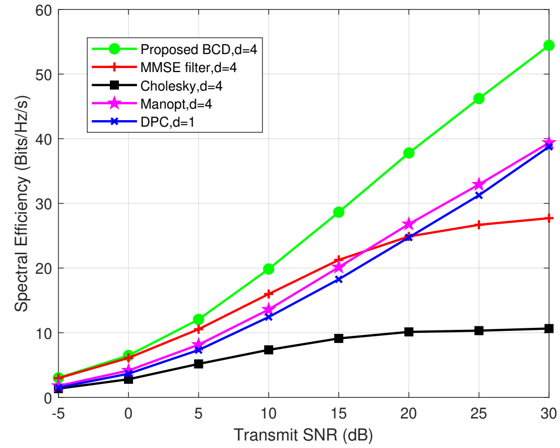

Fig. 1. Spectral efficiency versus transmit SNR under different schemes in the multi-user case.

The BS equipped with $N_{\mathrm{tx}} = 16$ transmit antennas sends $d = 4$ independent data streams to each downlink user. We assume that there are $K=4$ users, each equipped with $N_{\mathrm{rx}} = 4$ receive antennas. For simplicity, we set $\omega_k = 1, \forall k$. To compare "Proposed BCD" solution yielded by Algorithm 1, we consider the following benchmark schemes:

- *DPC* [13]: DPC is employed for the sum-rate maximization problem to pre-cancel the interference caused by radar and other users' signals at the transmitter. However, the DPC scheme proposed in [13] only applies to MISO systems. For a fair comparison, when user k is equipped with multiple antennas to receive a single data stream, the row with the largest 2-norm in $\mathbf{H}_k$ is selected for implementation of such a DPC scheme.
- *Manopt* [19]: Recall that constraint (8b) defines a complex Grassmann manifold for the feasible set of $\mathbf{F}$. Hence, problem (8) could be solved by the Riemannian gradient descent method delineated in Appendix A of our extended version [14].
- *MMSE filter* [12]: Here, MMSE is selected as the optimization objective instead of the weighted sum-rate (8a), and the transmit and receive beamformers are optimized in an alternating manner until convergence.
- *Cholesky*: The transmit beamforming matrix is set to $\mathbf{F} = \mathbf{L}$, where $\mathbf{L}$ is obtained by Cholesky decomposition of $\mathbf{R}_{\mathrm{des}}$ as defined in (14). Note that $\mathbf{L}$ also serves as the initial point of our proposed BCD algorithm.

Fig. 1 shows the the spectral efficiency (i.e., the achievable sum-rate under a unit bandwidth) obtained by different beamforming schemes under different transmit SNRs. It is observed that the proposed BCD-based scheme significantly outperforms the four benchmark schemes. The proposed BCD solution performs much better (e.g., yields a 97% higher special efficiency at 30 dB SNR) than the MMSE filter, especially in a high SNR regime. This is due to the fact that MMSE does not necessarily lead to the sum-rate maximization in most

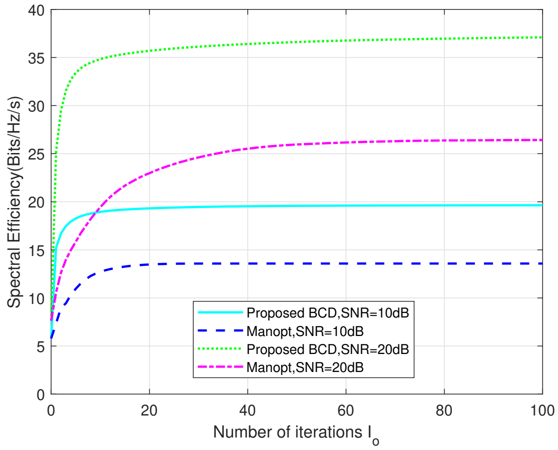

Fig. 2. Convergence behaviour under different schemes with $N_{\rm tx} = 16$, $d = 4$, $K = 4$, and $N_{\rm rx} = 4$.

cases. It is also clearly shown that the proposed BCD scheme exhibits superior performance compared to Manopt, achieving at least 40% higher spectral efficiency. Although, in principle, both the proposed BCD and the manifold optimization method can achieve stationary point solutions for problem (8) upon convergence, the simulation results verify that the proposed BCD algorithm can take advantage of the derived closed-form solutions for the subproblems in each step to facilitate faster convergence and yield a better-quality overall beamforming design than Manopt. In addition, a big gap (e.g., 5.1 times higher spectral efficiency at 30 dB SNR) between the proposed BCD solution and Cholesky decomposition clearly corroborates the substantial gain from our proposed BCD iterations. Compared with the DPC scheme that only supports $d = 1$ data stream transmission to each user, the proposed BCD solution supporting multi-stream transmissions per user could achieve a much (at least 41%) higher spectral efficiency, justifying that the proposed solution can effectively leverage the MIMO transmissions to benefit the communication capacity of DFRC systems. It is worth mentioning that implementation of the DPC scheme could dramatically raise the complexity of the multi-user MIMO systems and might even be computationally prohibitive in practice.

Fig. 2 shows the convergence behavior of the proposed method and Manopt under different transmit SNRs, where $d = 4$, $K = 4$, and $N_{\rm rx} = 4$. It is evident that the proposed BCD quickly converges within only a few iterations. Not only does it converge much faster than Manopt, but also yields beamforming solutions with much higher spectral efficiency, e.g., 45% higher at the 10 dB SNR, and 41% higher at the 20 dB SNR. The computational complexity and the implementation time of the proposed BCD approach is very low, comfirming its applicability in practical systems.

V. Conclusion

We developed beamforming design that maximizes the weighted sum of user rates under a prescribed transmit covariance constraint for MIMO DFRC systems. An efficient BCD-based algorithm was proposed to find a high-quality beamforming design with fast convergence and low computational complexity. Simulations demonstrated the superiority of the proposed schemes to the existing benchmarks, with at least 40% higher spectral efficiency under a multi-user MIMO setting in the medium-to-high SNR regime.

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