The Future of Network Science: Guiding the Formation of Networks

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Acknowledgement: ONR

Agenda

- Establish methods for “guiding” network formation
- Build a model of endogenous network evolution with incomplete information and learning
- Understand how learning and network formation co-evolve

Exogenous vs. Endogenous

Exogenously determined

- ❑ Predetermined by exogenous events

- Analyze ***given*** linking patterns
- How do agents learn about the ***exogenous environment***?
- How should information be ***disseminated***?
- Do agents in the network ***reach consensus?*** Are they ***herding***?

Endogenously evolving

- ❑ Determined by strategic choices of agents

- Analyze ***evolving*** linking patterns
- How do agents learn about ***each other***?
- How does information ***shape the network***?
- Do agents in the network ***cooperate and compete***?

Related Works - Network Formation

- Network formation under **complete information**
  - Homogeneous agents: [Jackson&Wolinsky'96], [Bala&Goyal'00], [Watts'01]
  - Heterogeneous agents: [Galeotti&Goyal'10], [Zhang&van der Schaar'12'13]
  - Known payoff parameters, no learning

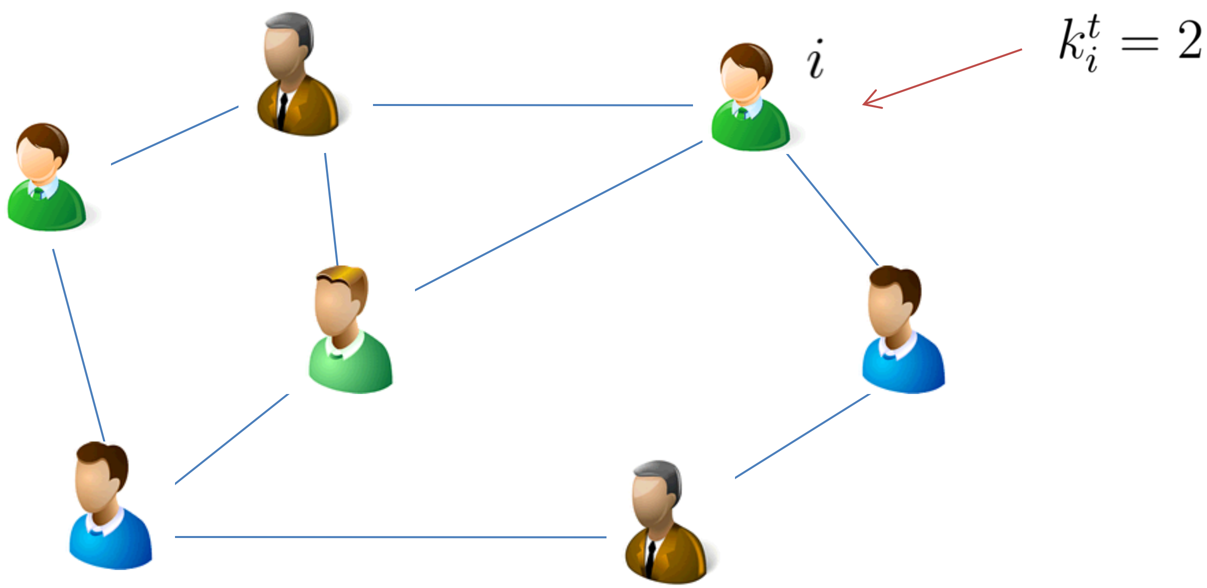
- Network formation under **incomplete information**

  [Song&van der Schaar'14]
  - Simplifying assumptions: know exactly after one interaction
  - No results about social welfare

- **New model needed!** Tractable model for computing social welfare, analyzing impact of learning and co-evolution of network structures, ability to guide network formation

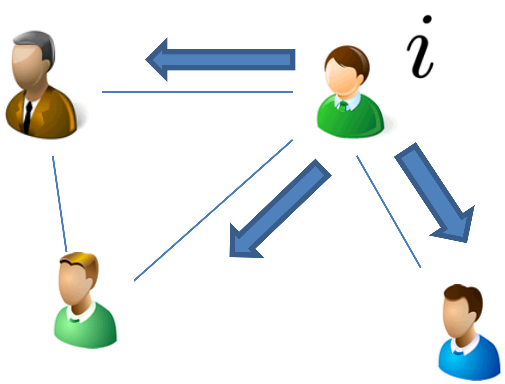
Network Model

- Infinite horizon continuous time
  - Interactions are on-going
- N agents, initially linked according to G^0
  - Physical/geographical/communication connection constraints
  - Planned
- Network evolves over time G^t
  - $k_i^t = \sum_j g_{ij}^t$: number of links (neighbors) of agent i at time t



Agent Quality

- Agent *i* has quality q_i
  - Unknown a priori
  - Prior belief: drawn from a normal distribution $\mathcal{N}(\mu_i, \sigma_i^2)$
  - Different agents, different distributions!
    - Good agents, bad agents
- Agent *i* sends (flow) benefit to agents to whom *i* is linked
  - Benefit = quality + noise
  - Modeled using Brownian motion diffusion



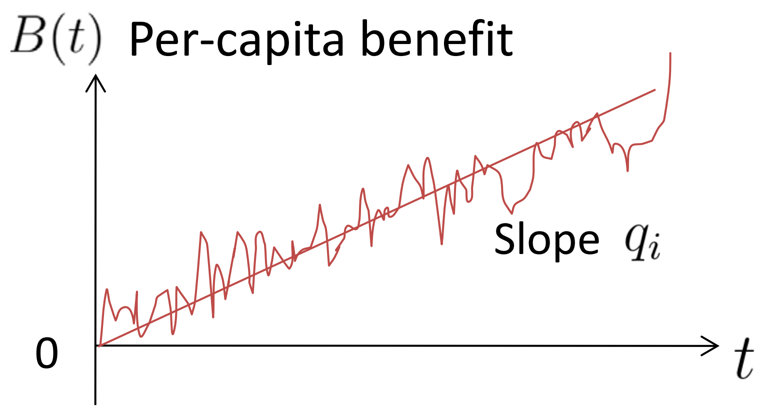
$$dB_i(t) = q_i dt + (k_i^t \tau_i)^{-1/2} dZ(t)$$

Per-capita benefit sent by agent i up to time t

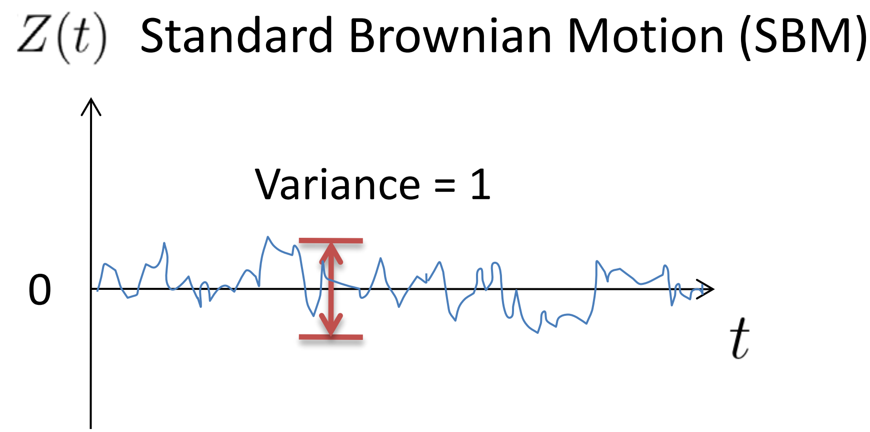
Noisy Benefit Flow

$$dB_i(t) = \underbrace{q_i dt}_{\text{Benefit reflecting the true quality}} + \underbrace{(k_i^t \tau_i)^{-1/2} dZ(t)}_{\text{Noise term}}$$

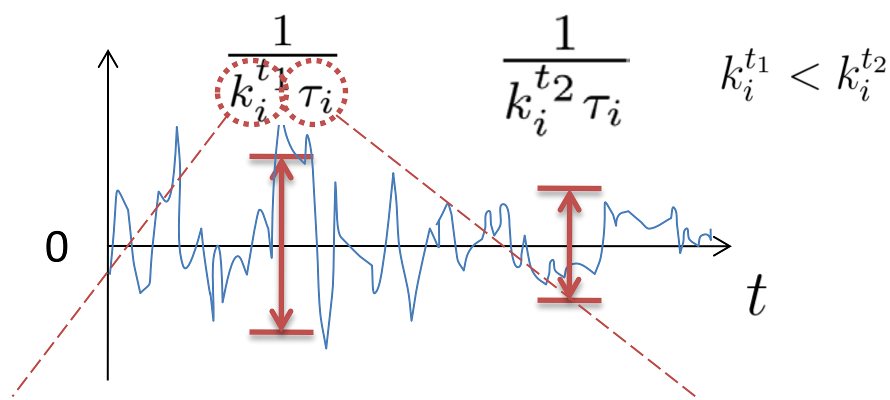
Without noise



With noise



Noise: "Modulated" SBM



Number of current neighbors

Base precision of an agent

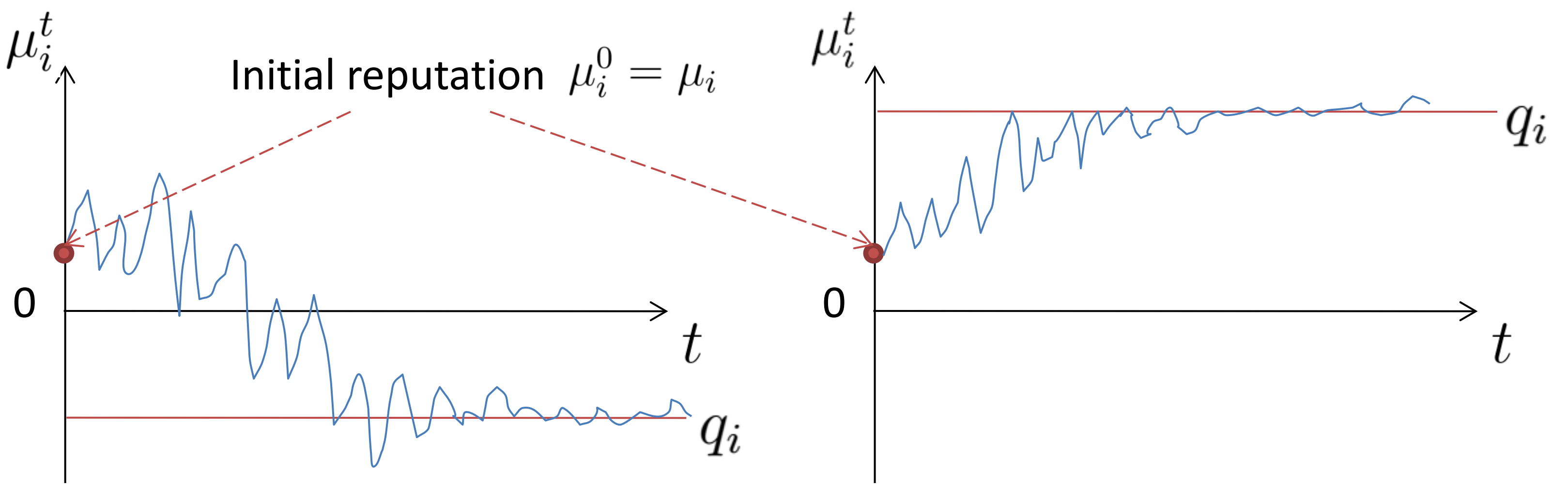
Larger base precision & more neighbors
→ Less noise

Reputation

- Expected quality conditional on observed benefit history

$$\mu_i^t = E[q_i | \{b_i^t\}_{t'=0}^t]$$

  - Updated according to Bayes rule (learning)
  - **Suppose** always connected and generating benefit flow



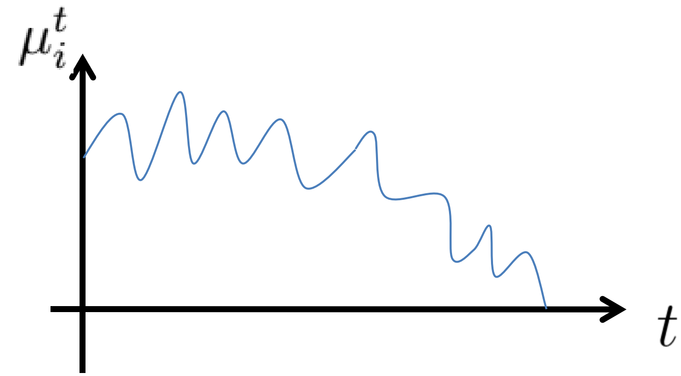
Low quality agent will be learned to be of low quality

High quality agent will be learned to be of high quality

Network Evolution

Agents are myopic

- Goal: Maximize instantaneous utility
  - Connect $\mu_i^t > 0$
  - Disconnect $\mu_i^t \leq 0$

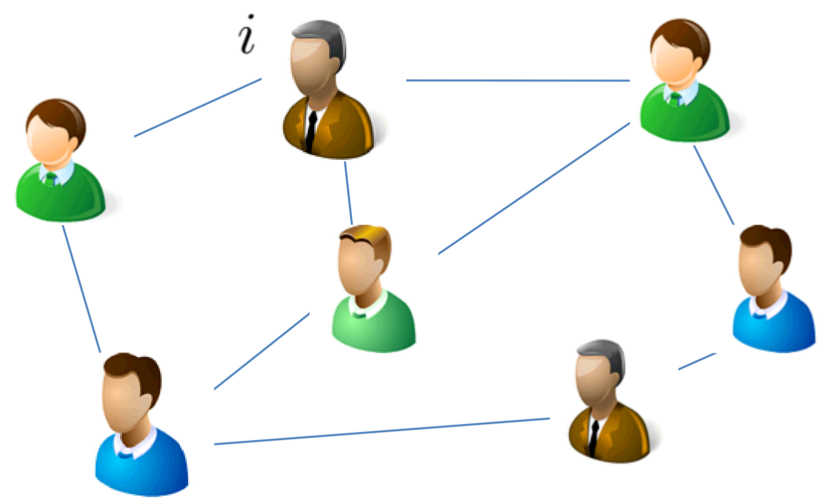


Agent *i*'s neighbors cut off links with Agent *i*

Agent *i* gets ostracized from the network

Learning about Agent *i*'s neighbors slows down (since they have fewer links)

Process continues and more agents may be ostracized

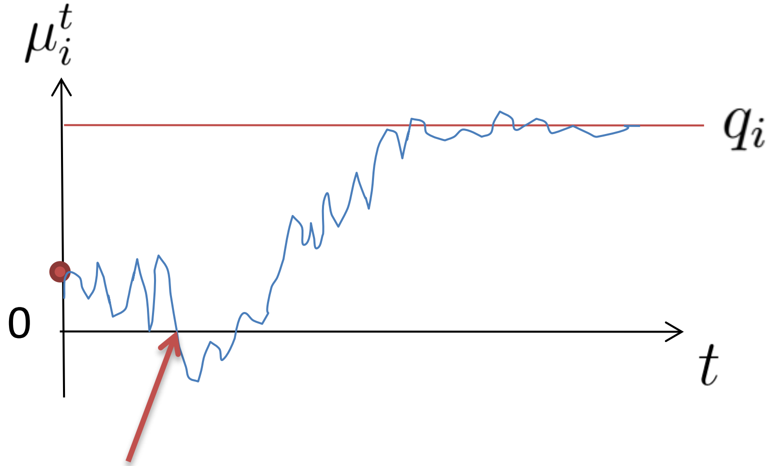


Stability

Stability = Network does not change over time

Theorem. From any initial configuration, convergence to a stable network always occurs in finite time

- Low quality agents
  - Always be learned to be low quality → will always be ostracized (never in a stable network)
- High quality agents
  - If learned to be high quality → will stay in the network forever
  - If believed to be low quality (by accident) → will be ostracized

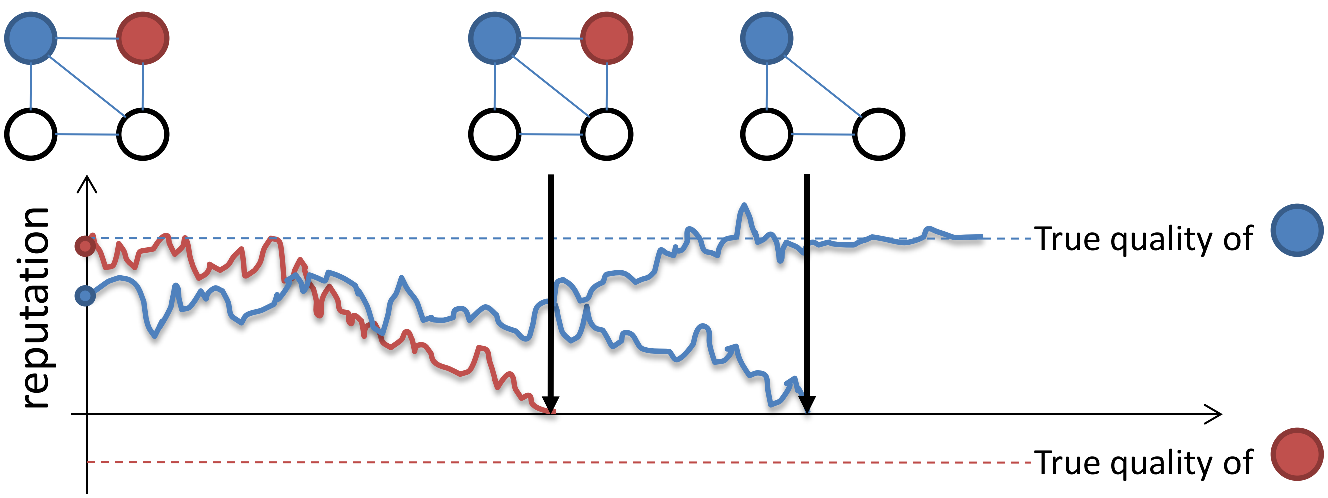


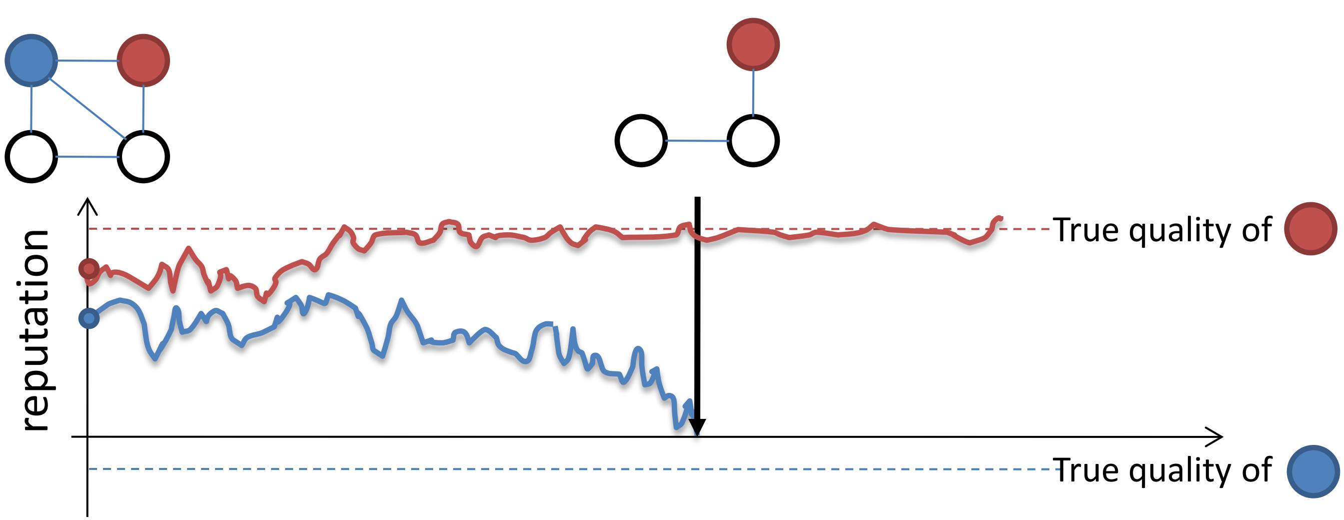
Ostracized by accident

MANY possible stable networks!

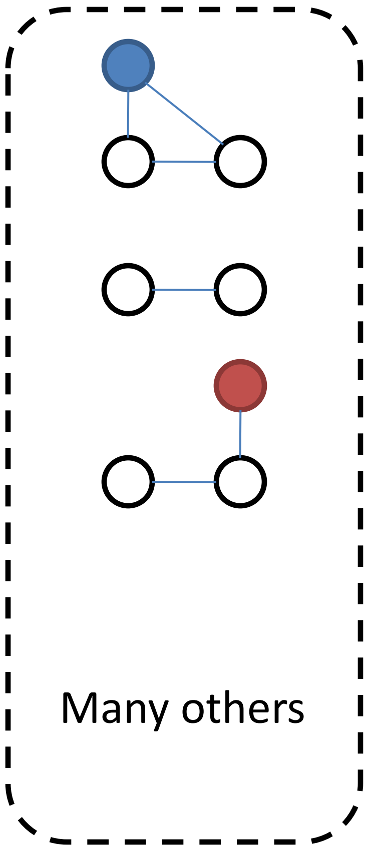
Which one emerges? Random! Different probabilities

Random Evolution

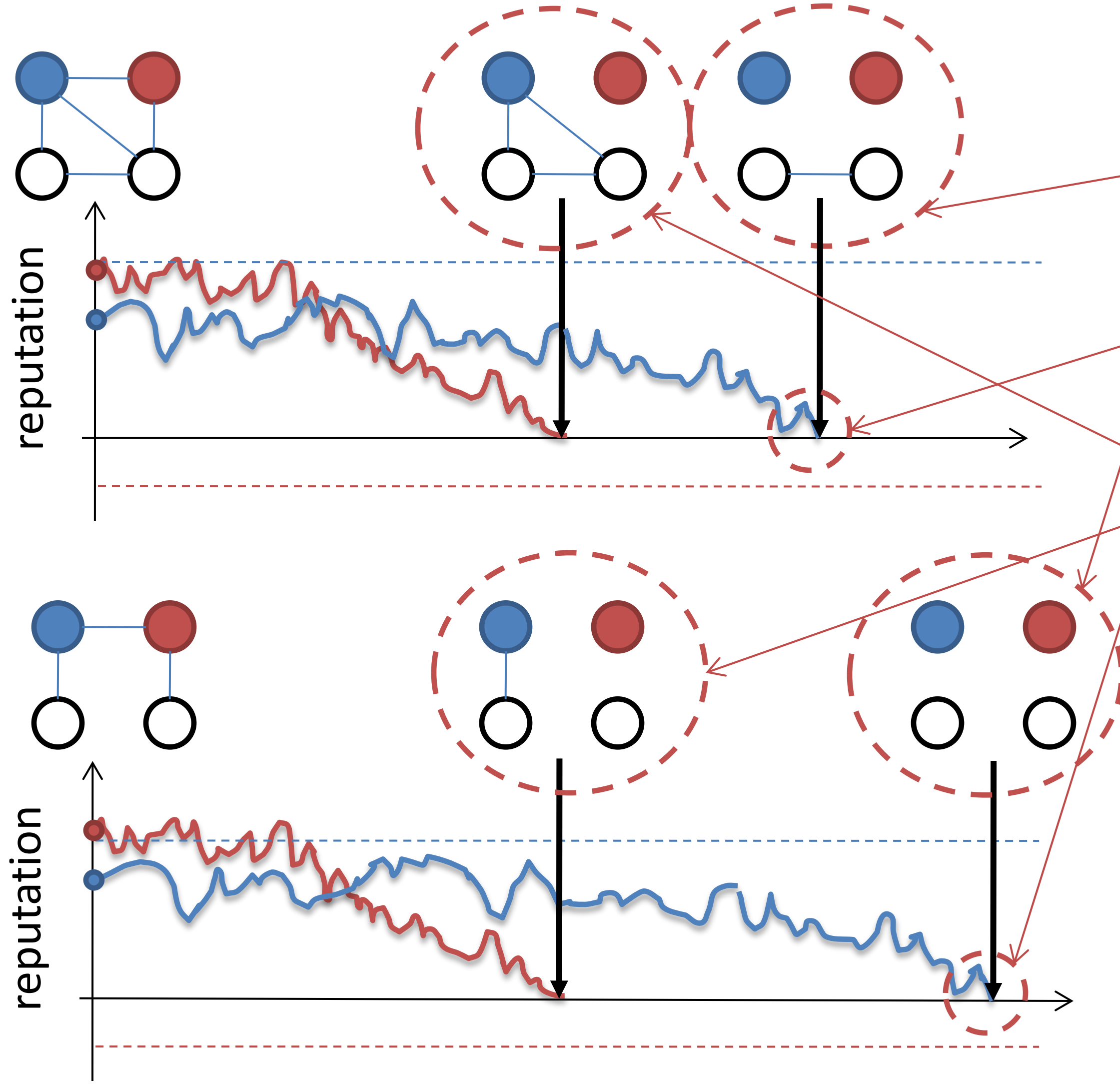




Stable Networks



Initial Network Matters!



Different stable networks

Different time when it becomes stable

Different intermediate networks

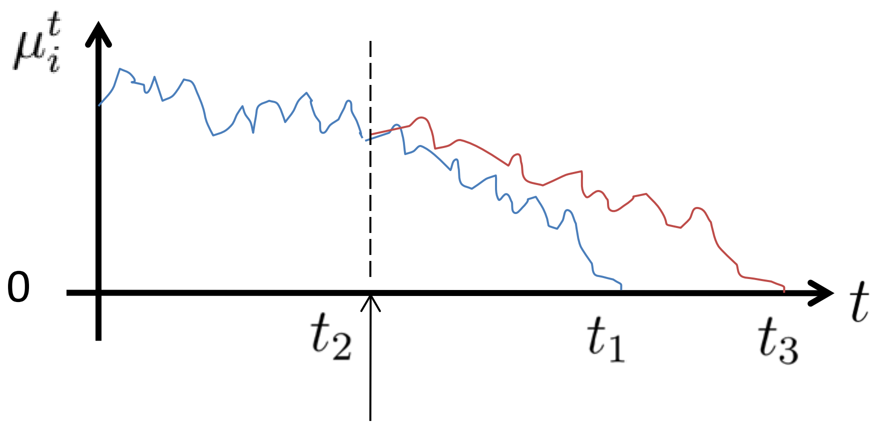
Initial network should be carefully planned!

Ostracism

Proposition. The probability that agent *i* is ostracized in the long run is *independent* of the initial network.

(The time it takes for agent *i* to be ostracized is *not* independent of the initial network.)

– Scaling effect:



One neighbor is ostracized
→ Fewer links

Changes when the hitting occurs

Does not change whether the hitting occurs

Does not change whether the agent stays in the stable network in this realization

What networks can emerge and be stable?

- Ex-ante probability that agent i with initial reputation μ_i is never ostracized

$$\int_0^\infty (1 - \exp(-\frac{2}{\sigma_i^2}\mu_i q_i))\phi\left((q_i - \mu_i)\frac{1}{\sigma_i}\right) dq_i$$

Theorem. Beginning from an initial configuration G0, a network G can emerge and be stable with positive probability if and only if G can be reached from G0 by sequentially ostracizing agents

Guiding network formation

- Planner’s goal
  - Maximize long-term welfare (discount factor ρ)

- What does the planner know?
  - The initial reputations of agents
  - *Not* the true quality of agents

- What can the planner do?
  - Set an initial connectivity of the network

Social Welfare

- How to define social welfare?
  - Path of network evolution is random
    - **It is not only about the limit stable network, but also about the intermediate networks that matter**
  - The "in expectation" perspective
    - Initial reputation (Prior belief about agents' quality)
    - Initial network topology

Definition: *Ex ante* discounted long-term sum benefit

$$W = \boxed{\int_{q_1=-\infty}^{\infty} \dots \int_{q_N=-\infty}^{\infty}} \sum_{i,j} \int_0^{\infty} \underline{e^{-\rho t}} q_j \underline{P(L_{ij}^t | \boldsymbol{q}, G^0)} \phi(\frac{q_N - \mu_N}{\sigma_N}) dq_N \dots \phi(\frac{q_1 - \mu_1}{\sigma_1}) dq_1$$

Discounting

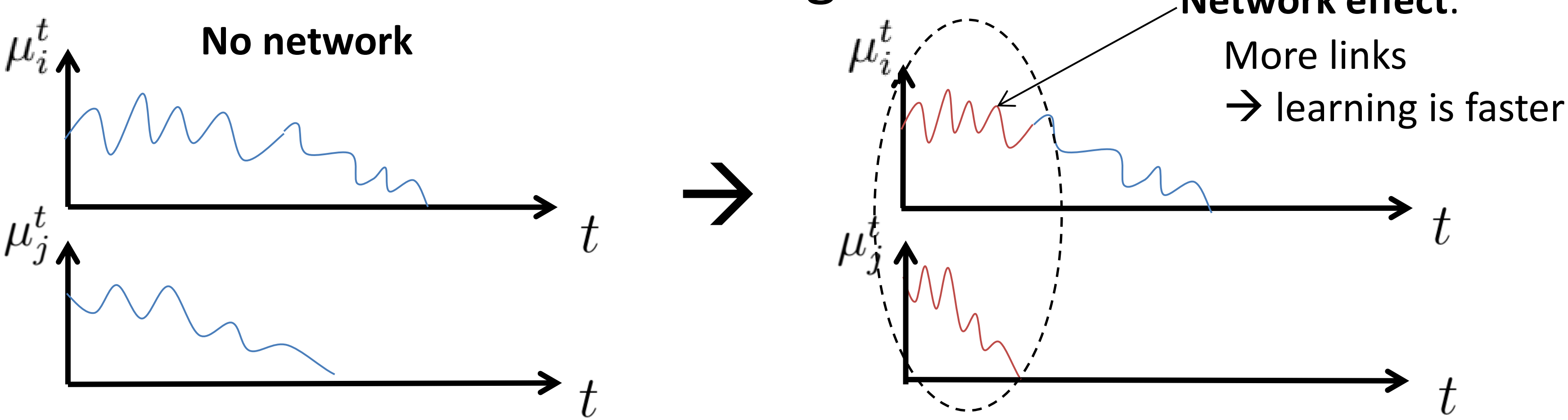
Survival probabilities of links

Expectation using prior belief

Extremely difficult to compute: numerous conditional probabilities

Ex Post → Ex Ante

- Network effect: the scaling effect



Compute distributions

Reconstruct realization
Compute *ex post* welfare

Theorem. The ex ante social welfare can be computed in a closed form as follows

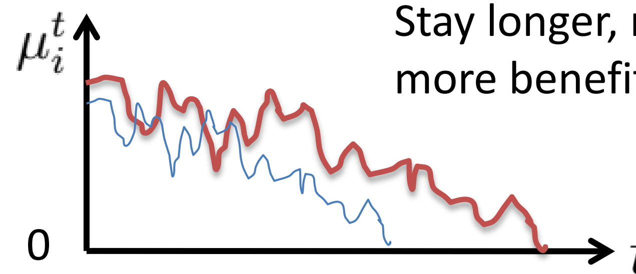
$$W = \boxed{E_{\hat{\varepsilon}}} \sum_i \left(\frac{1 - e^{\boxed{-\rho M_i(\boldsymbol{t})}}}{\rho} \sum_{j: g_{ij}^0 = 1, t_j = \infty} \frac{\mu_j}{\boxed{P(S_j)}} \right)$$

How learning affects individuals' welfare?

$$dB_i(t) = q_i dt + (k_i^t \tau_i)^{-1/2} dZ(t)$$

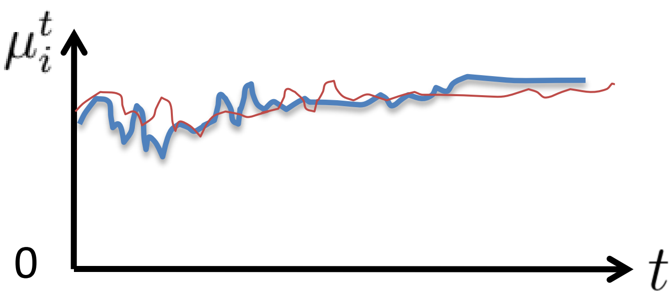
Base precision of an agent: information sending speed

- Low quality agents
  - Want to be learned about more slowly

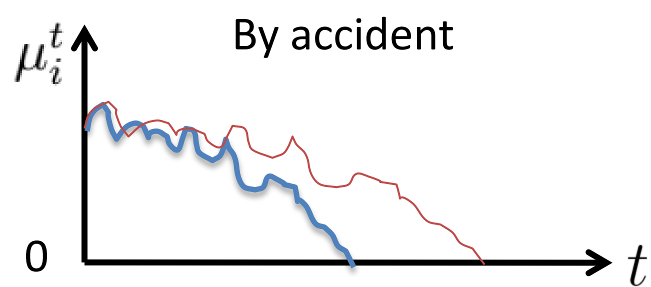


Stay longer, receive more benefit

- High quality agents
  - Want to be learned about more quickly?



Not affected in this case



Worse off in this case

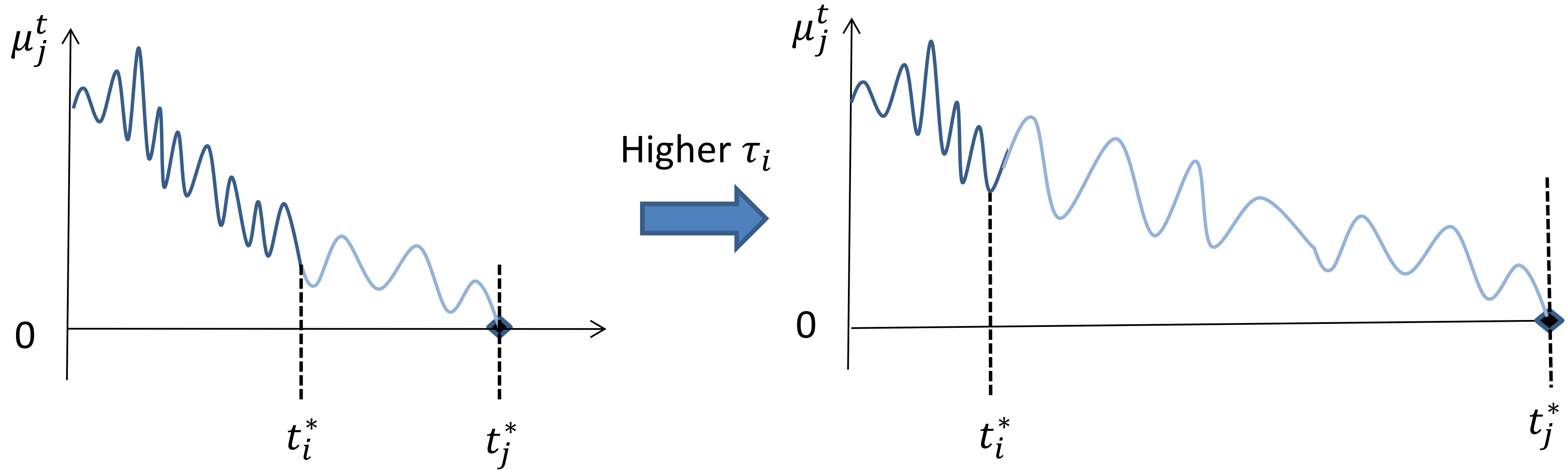
High quality agents also want to be learned about more slowly

Impact of Learning Speed on Welfare

Theorem. For any initial network, each agent i's welfare is ***decreasing*** in its base precision τ_i.
Further, multiplying all agents' base precisions by the same factor $d > 1$ decreases the total *ex ante* social welfare.

Theorem. For any initial network without cycles, increasing any agent i's base precision τ_i ***increases*** the welfare of each of i's neighbors.

Increasing Agent i's Precision helps its Neighbor



Neighbor j's hitting time increases!

Agent j gets more benefits from network!

Optimal Initial Connection

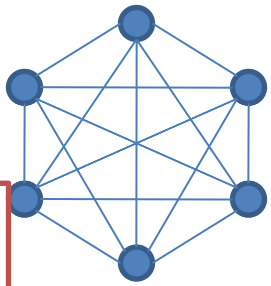
- Depends on planner's patience ρ

- Completely impatient – only the initial network matters
- Completely patient – only the limit stable network matters

- These cases are NOT very interesting
- Intermediate patience $0 < \rho < 1$?

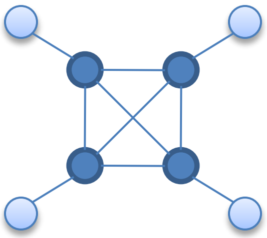
Optimal Initial Network

- Fully connected network



Theorem. A fully connected initial network is optimal if all prior mean qualities are sufficiently high (depending on ρ)

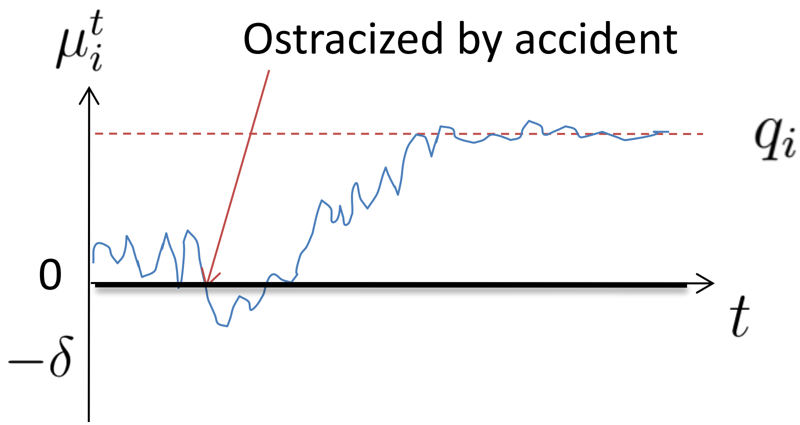
- Core-periphery network
  - Heterogeneous agents: two levels μ_H μ_L

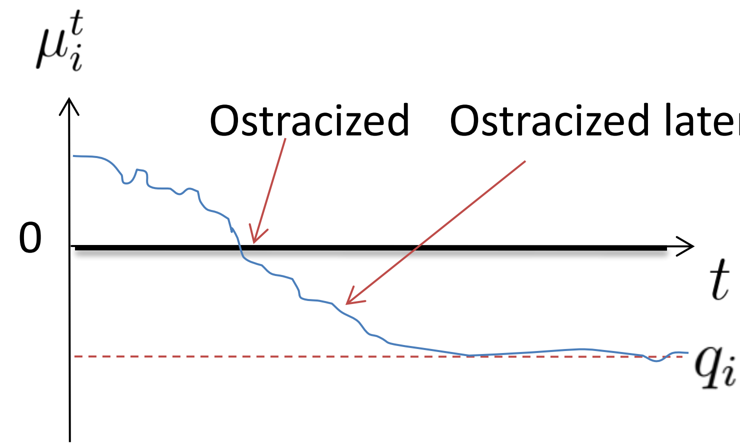


Theorem. A core-periphery initial network is optimal if μ_H is sufficiently higher than μ_L (depending on ρ)

- Why?
  - High quality in the core → learned more quickly
  - Low quality in the periphery → less harm

Encouraging experimentation





Theorem. (1) $\exists\ \underline{\delta}$ s.t. $W(\delta) > W(0)$ for all $\delta > \underline{\delta}$

(2) $\delta^* = \arg\max_\delta W(\delta)$ exists and is finite.

– Experimentation is good for social welfare

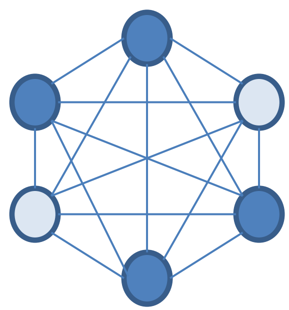
– Cannot be too tolerant to bad behaviors

– δ^* is computable!

Incorporating Agent Entry

- Our model can be tractably extended to allow agents to *enter* the network over time
  - E.g. a firm does not hire all workers immediately, but introduces them in a sequential order

Initial Network



$t = 1$

$t = 2$

Reconstruct realization based on agent entry time

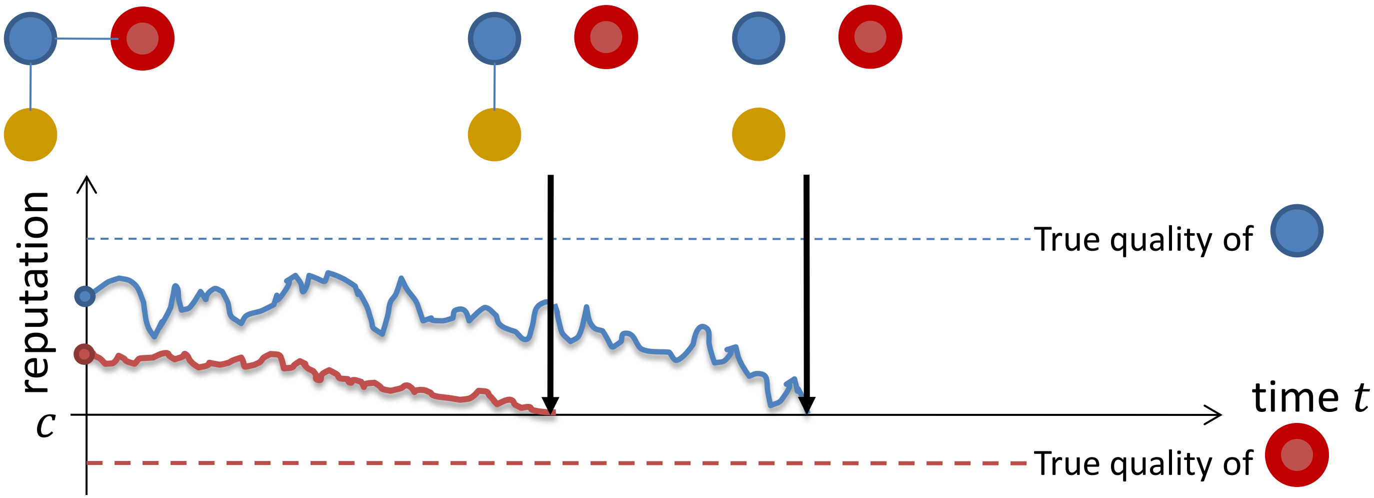
Theorem. The ex ante social welfare can be computed as follows

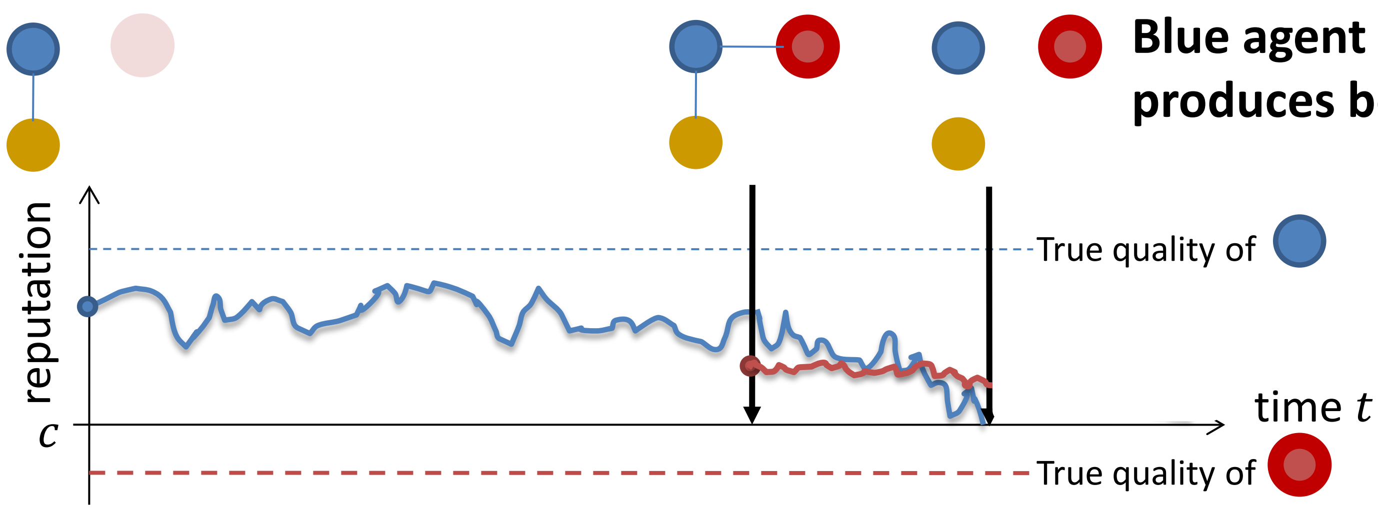
$$W = E_{\hat{\varepsilon}} \sum_i \left(\sum_{j: g_{ij}^0 = 1, t_j = \infty} \frac{e^{\boxed{-\rho S_{ij}(t)}} - e^{\boxed{-\rho F_{ij}(t)}}}{\rho} \frac{\mu_j}{P(S_j)} \right)$$

Delaying Entry Can Improve Welfare

- By allowing agents to enter later, social welfare can be improved in certain networks
- Agents can have more time to cement their reputations without getting ostracized from the network as quickly

Delaying Entry Can Improve Welfare





Blue agent receives and produces benefits for longer!

Conclusions

- The first model of endogenous network evolution with incomplete information and learning
  - Rigorous characterization of learning and network co-evolution
  - Understanding emergent behaviors of strategic agents
- Guiding network formation
  - Planning initial configuration
  - Encouraging experimentation
  - Deciding “entry” times of agents